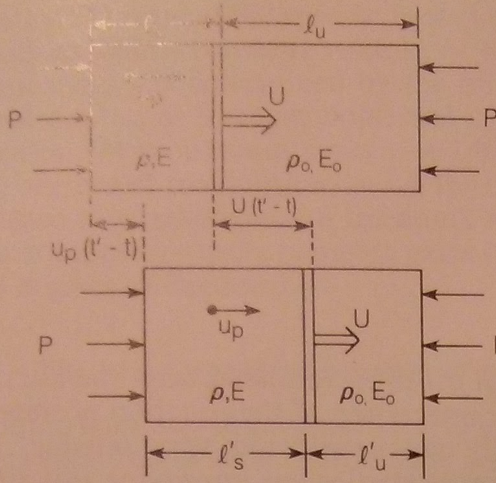

Fig. A1.1 Free body diagrams of a shock wave passing through a mass of material at times t (above) and t' (below).

equation (3.4.2) in the text,

$$P - P_0 = \rho_0 U u_p \qquad \text{(AI.2.3)}$$

AI.3. Energy conservation

Like momentum, the total energy in the block at time t is not equal to that at time t' because the applied forces do work on the system. This work is equal to the force times the distance through which it acts. Since the displacement of the unshocked end of the block is zero, the total energy gained between t and t' is thus $PAu_p\,(t' - t)$, equal to the force PA on the shocked end of the block times the distance $u_p(t' - t)$ through which it acts.

The total energy $E_{tot}\,(t)$ in the block at time t is the sum of the internal energies in the shocked and unshocked portions and the kinetic energy in the shocked portion:

$$E_{tot}(t) = \rho_0 l_u E_0 A + \rho l_s E A + 1/2\, \rho l_s u_p^2 A \qquad \text{(AI.3.1a)}$$

Similarly, at time t' the total energy is

$$E_{tot}(t') = \rho_0 l_u' E_0 A + \rho l_s' E A + ½\, \rho l_s' u_p^2 A \qquad \text{(A.3.1b)}$$

Energy conservation thus requires

$$E_{tot}(t') - E_{tot}(t) = PAu_p\,(t' - t) \qquad \text{(A1.3.2)}$$

Substituting Equations AI.3.1a and AI.3.1b into Equation AI.3.2, cancel A through as before, substitute Equation AI.1.1a for l'_u and Equation AI.1.1b for l'_s, and simplify. The common factor $(t' - t)$ may then be canceled to obtain

$$-\rho_0 E_0 U + \rho E(U - u_p) + ½\, \rho u_p^2\,(U - u_p) = Pu_p \qquad \text{(AI.3.3)}$$

Now replace $\rho(U - u_p)$ by $\rho_0 U$ using the first Hugoniot equation (AI.1.4) to obtain

$$\rho_0 U(E - E_0) + ½\, \rho_0 u_p^2 U = Pu_p \qquad \text{((AI.3.4)}$$

We then proceed using two auxiliary equations that can be readily derived from the first two Hugoniot equations (AI.1.4) and (AI.2.3) by eliminating either U or u_p, respectively, between the two equations:

$$u_p = \sqrt{(P - P_0)\,(V - V_0)} \qquad \text{(AI.3.5a)}$$

and

$$U = 1/\rho_0 \sqrt{(P - P_0)/(V - V_0)} \qquad \text{(AI.3.5b)}$$

where $V = 1/\rho$ and $V_0 = 1/\rho_0$ are the specific volumes of the shocked and unshocked material, respectively. Substituting Equation AI.3.5a for u_p and AI.3.5b for U in Equation AI.3.4, canceling the common factor $\sqrt{(P - P_0)}$, and rearranging,

$$E - E_0 = ½\,(P + P_0)\,(V_0 - V) \qquad \text{(AI.3.6)}$$

which is the third, and final, Hugoniot equation (3.4.3) of the text.