GEOS 38600/ GEOS 28600

Lecture 12

Monday 20 Feb 2017

Fluvial sediment transport: introduction

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REVIEW OF REQUIRED READING (SCHOOF & HEWITT 2013)

TURBULENT VELOCITY PROFILES, INITIATION OF MOTION

BEDLOAD, RIVER GEOMETRY

$$\frac{\partial \tau_{ij}}{\partial x_j} - \frac{\partial p}{\partial x_i} + \rho g_i = 0,$$

Re << 1 → inertial forces unimportant → Stokes flow / creeping flow:

Ice sheets can have multiple stable equilibria for the same external forcing, with geologically rapid transitions between equilibria

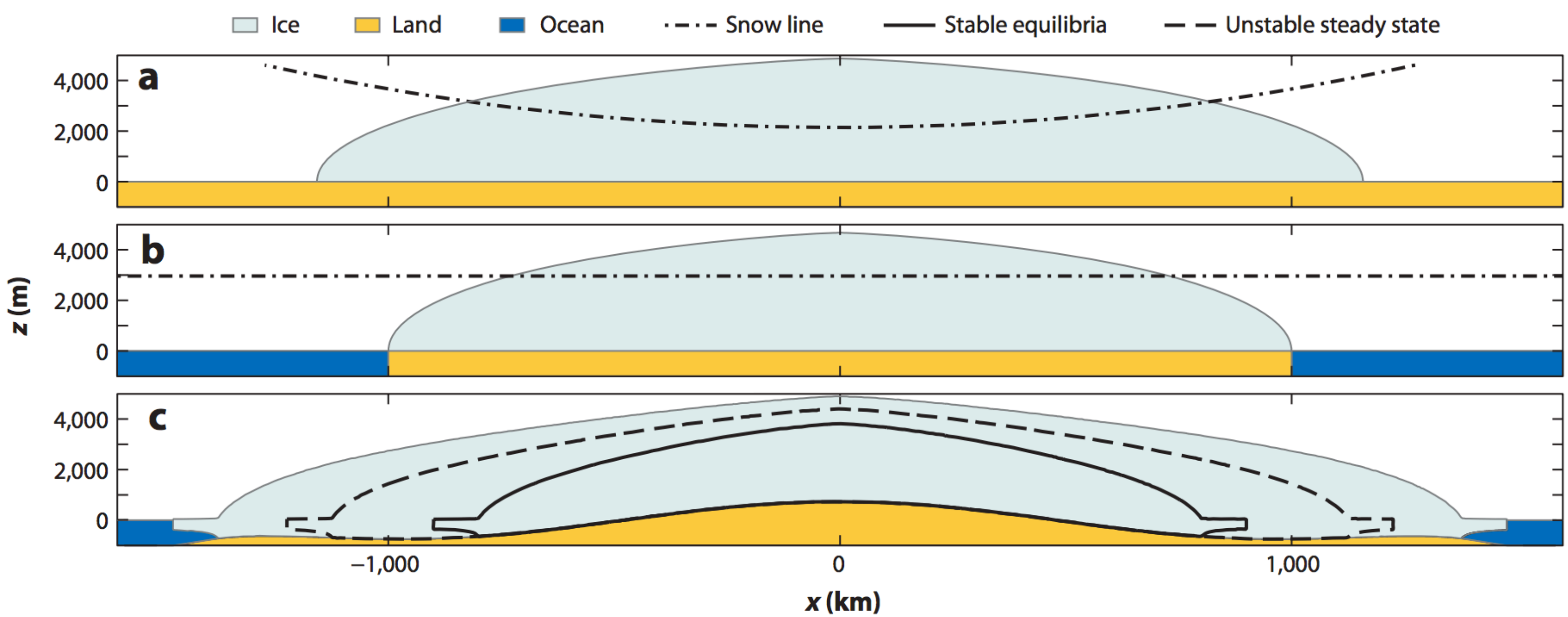

Figure 2

Three different flavors of steady shallow ice sheets. (*a*,*b*) The surface experiences net accumulation when above the snow line (*dotted-dashed line*) and net melting below. In panel *a*, there is more melting with distance from the origin, and this determines the steady-state margin position. In panel *b*, the snow line is flat, and ice covers the entire continental land mass. In panels *a* and *b*, the ice-free state is also a viable steady state. (*c*) A marine ice sheet (Section 7), with no surface melting but with multiple equilibria generated by an overdeepened bed. Solid lines correspond to stable equilibria, and the dashed line is an unstable steady state.

Schoof & Hewitt 2013

Key points from today's lecture

- Critical Shields stress
- Differences between gravel-bed vs. sand-bed rivers
- Discharge-width scaling

Prospectus: fluvial processes

- Today: overview, hydraulics, initiation of motion, channel width adjustment.
- Channel long-profile evolution.
- Mountain belts.
- Final lectures: landscape evolution (including fluvial processes.)

This section of the course draws on courses by W.E. Dietrich (Berkeley), D. Mohrig (MIT → U.T. Austin), and J. Southard (MIT).

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Hydraulics and sediment transport in rivers:

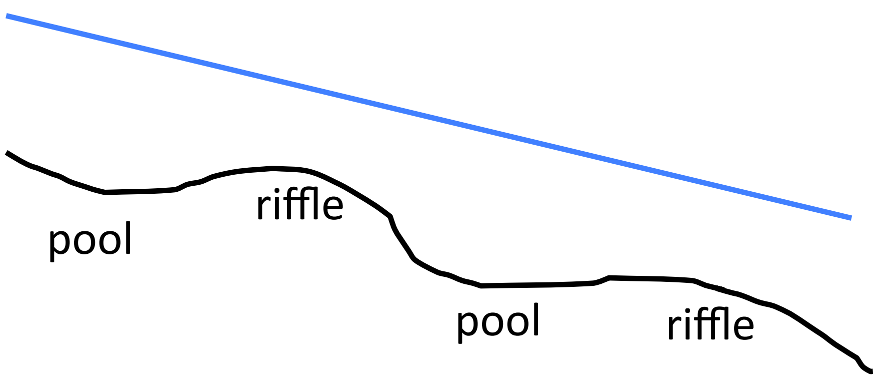
1) Relate flow to frictional resistance so can relate discharge to hydraulic geometry.

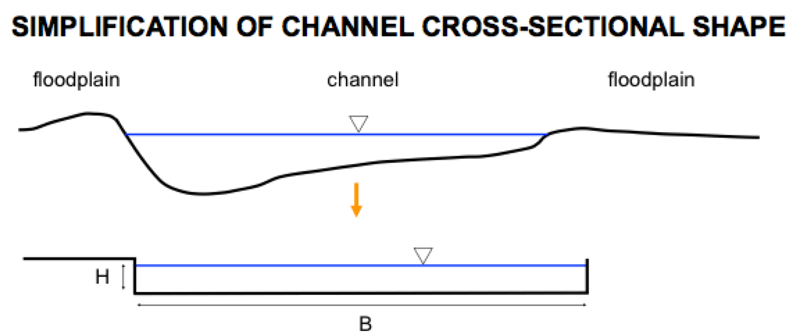
2) Calculate the boundary shear stress.

Simplified geometry: average over a reach (12-15 channel widths).

→ we can assume accelerations are zero.

→ this assumption is better for flood flow (when most of the erosion occurs)





Parker Morphodynamics e-book

The assumption of no acceleration requires that gravity balances pressure gradients.

$$\tau_{zx} = \rho g h \sin\theta$$

averaging over 15-20 channel widths forces the water slope to ~ parallel the basal slope

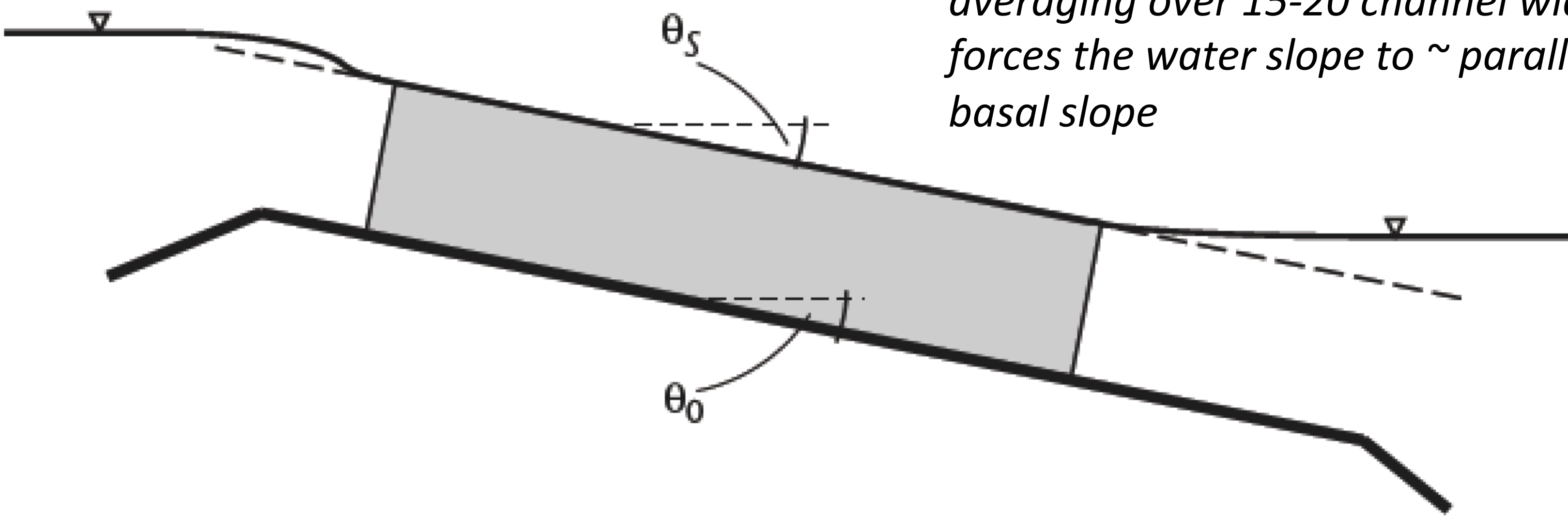
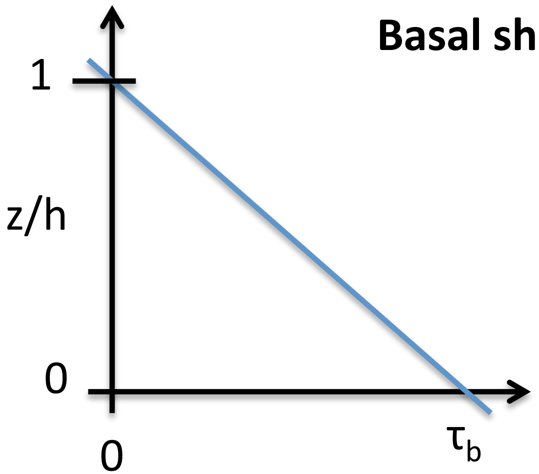


Figure 6.2 Idealized development of uniform flow in a channel of constant slope, θ_0, geometry, and bed material connecting two reservoirs. The shaded area is the region of uniform flow, where the downstream component of gravity is balanced by frictional resistance and the water-surface slope θ_S equals θ_0.

Dingman, chapter 6

Basal shear stress, frictional resistance, and hydraulic radius



$\tau_{zx} = \rho g h \sin\theta$

At low slope (S, water surface rise/run), $\theta \sim \tan\theta \sim \sin\theta$

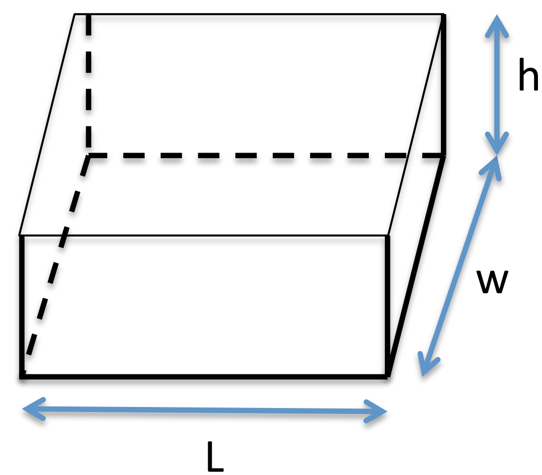
$\tau_b = \rho g h\, S$

<u>Frictional resistance:</u>

Boundary stress = $\rho g h \sin\theta\, L\, w$
Frictional resistance = $\tau_b\, L\, (w + 2\, h)$

$\rho g h \sin\theta\, L\, w = \tau_b\, L\, (w + 2\, h)$

$\rightarrow \tau_b = \rho g h\, (\, w\, /\, (w + 2\, h\,)\,) \sin\theta$



Define hydraulic radius, $R = hw\, /\, (w + 2\, h)$
$\rightarrow \tau_b = \rho g h R \sin\theta$

In very wide channels, $R \rightarrow h$ ($w \gg h$)

Law of the wall, recap:

Rivers (Re >> 1, fully turbulent):

$\tau_{zx} = \rho\, K_T\, (du/dz)$

eddy viscosity, "diffuses" velocity

Glaciers (Re << 1):

$\tau_{zx} = \mu(T,\sigma)\, (du/dz)$

Properties of turbulence:
Irregularity
Diffusivity
Vorticity
Dissipation

From empirical & theoretical studies:

$K_T = (k\, z)^2\, (du/dz)$ *(where k = 0.39-0.4 = von Karman's constant)*

$\rightarrow \tau_B = \rho(k\, z)^2\, (du/dz)^2$

$\rightarrow (\tau_B\ /\rho\ )^{1/2} = k\, z\, (du\ /\ dz) = u^* =$ "shear velocity"

Memorize this.

$\rightarrow (\rho\, g\, h\, S\ /\rho\ )^{1/2} = u^* = (\ g\, h\, S\ )^{1/2}$

Now $u^* = k\, z\, (du\ /\ dz)$

Separate variables: $du = (u^*\ /\ k\, z\ )\, dz$

Integrate: $u = (u^*/k)\, (\ln z + c)$. For convenience, set $c = -\ln(z_0)$

"law of the wall"

Then, $u = (u^*/k)\, \ln(z/z_0)$

when $z = z_0$, $u = 0$ m/s.

(explained on next slide)

Calculating river discharge, Q (m^3s^{-1})

$u = (u^*/k) \ln (z/z_0)$

"law of the wall"

z_0 is a length scale for grain roughness varies with the size of the bedload. In this class, use $z_0 = 0.12\ D_{84}$, where D_{84} is the 84[th] percentile size in a pebble-count (100[th] percentile is the biggest).

$Q = \langle u \rangle\ w\ h$ *brackets denote vertical average*

$$\langle u \rangle = \int_{z_0}^{h} u(z)\, dz \quad (1/(h-z_0))$$

$$\langle u \rangle = (u^*/k)\ (z0 + h\ (\ \ln(\ h\ /\ z_0\ ) - 1\ )\ )\ (1/\ (h - z_0))$$

$h >> z_0$:

$$\langle u \rangle = (u^*/k) \quad (\ \ln(\ h\ /\ z_0\ ) - 1\ )$$

$$\langle u \rangle = (u^*/k) \ln (\ h\ /\ e\ z_0)$$

$$\langle u \rangle = (u^*/k) \ln (0.368\ h\ /\ z_0)$$

typically rounded to 0.4

Extending the law of the wall through the flow is a rough approximation – do not use this for civil-engineering applications. This approach does not work at all when depth → clast grainsize.

Drag coefficient for bed particles:

3 alternative methods

$\rightarrow \tau_B = \rho g R S = C_D\, \rho <u>^2 / 2$

$<u> = (2g\, R\, S / C_D)^{1/2}$ — $(2g / C_D)^{1/2} = C$ = Chezy coefficient

$<u> = C (R\, S)^{1/2}$ — Chezy equation (1769)

$<u> = (8\, g / f)^{1/2} (R\, S)^{1/2}$ — f = Darcy-Weisbach friction factor

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$<u> = R^{2/3} S^{1/2} n^{-1}$ — n = Manning roughness coefficient

Most used, because lots of investment in measuring *n* for different objects

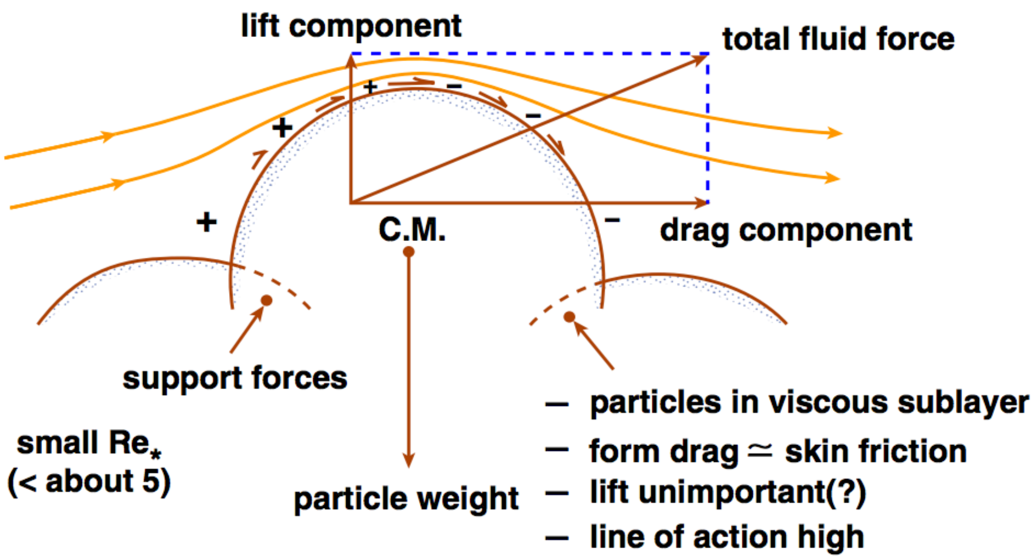
$0.025 < n < 0.03$ ----- Clean, straight rivers (no debris or wood in channel)
$0.033 < n < 0.03$ ----- Winding rivers with pools and riffles
$0.075 < n < 0.15$ ----- Weedy, winding and overgrown rivers
$n = 0.031(D_{84})^{1/6}$ ---- Straight, gravelled rivers

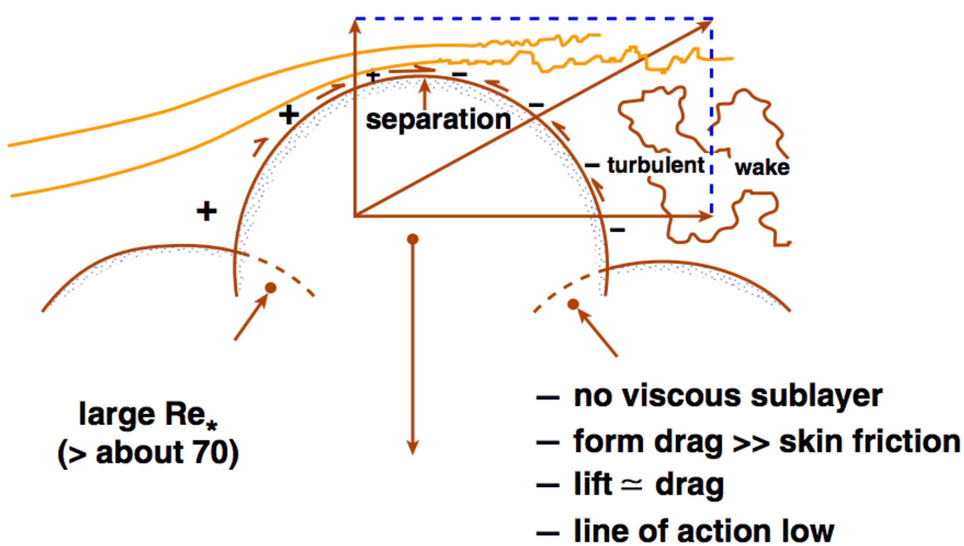
In sand-bedded rivers (e.g. Mississippi), form drag due to sand dunes is important.

In very steep streams, supercritical flow may occur:

supercritical flow

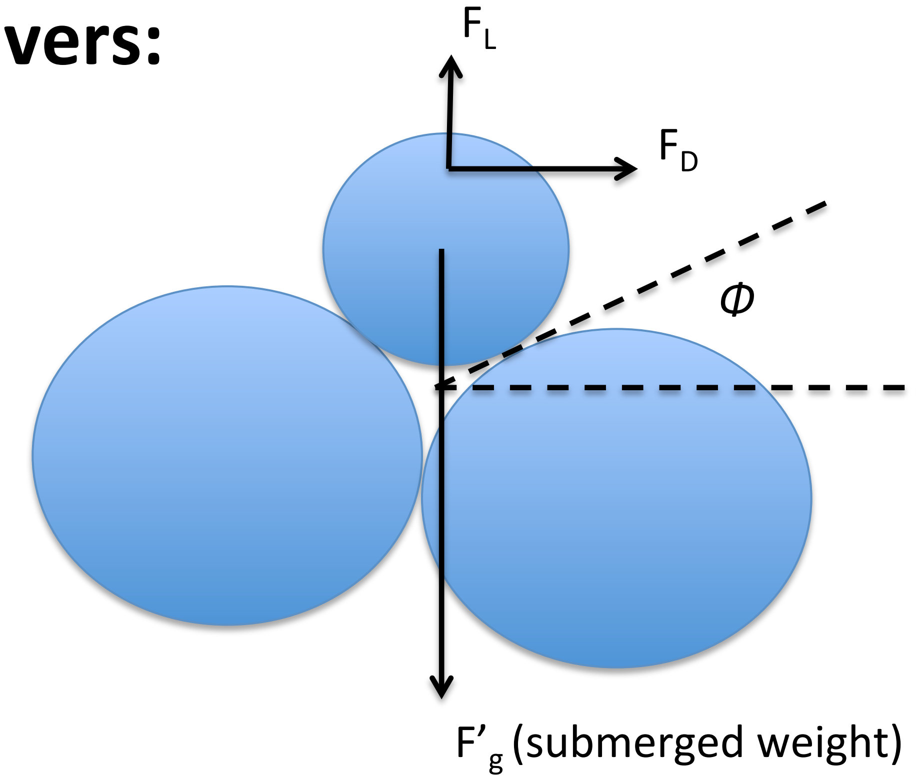
Froude number $Fr\ \# = <u>/(gh)^{1/2} > 1$





John Southard

Sediment transport in rivers: (Shields number)



At the initiation of grain motion,

$$F_D = (F'_g - F_L) \tan \Phi$$

$$\rightarrow F_D/F'_g = \frac{\tan \Phi}{1 + (F_L/F_D) \tan \Phi}$$

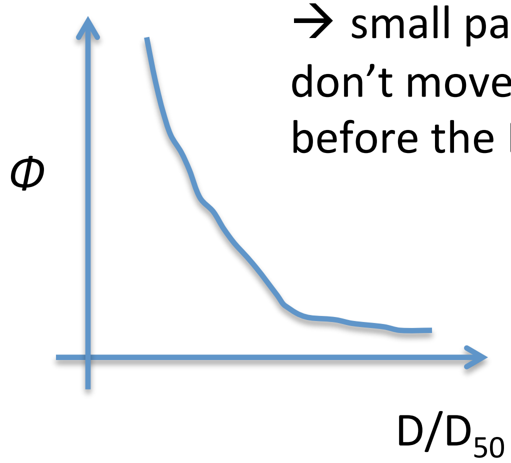
$$\approx \frac{\tau_c D^2}{(\rho_s - \rho) g D^3}$$

$$= \frac{\tau_c}{(\rho_s - \rho) g D} = \tau_*$$

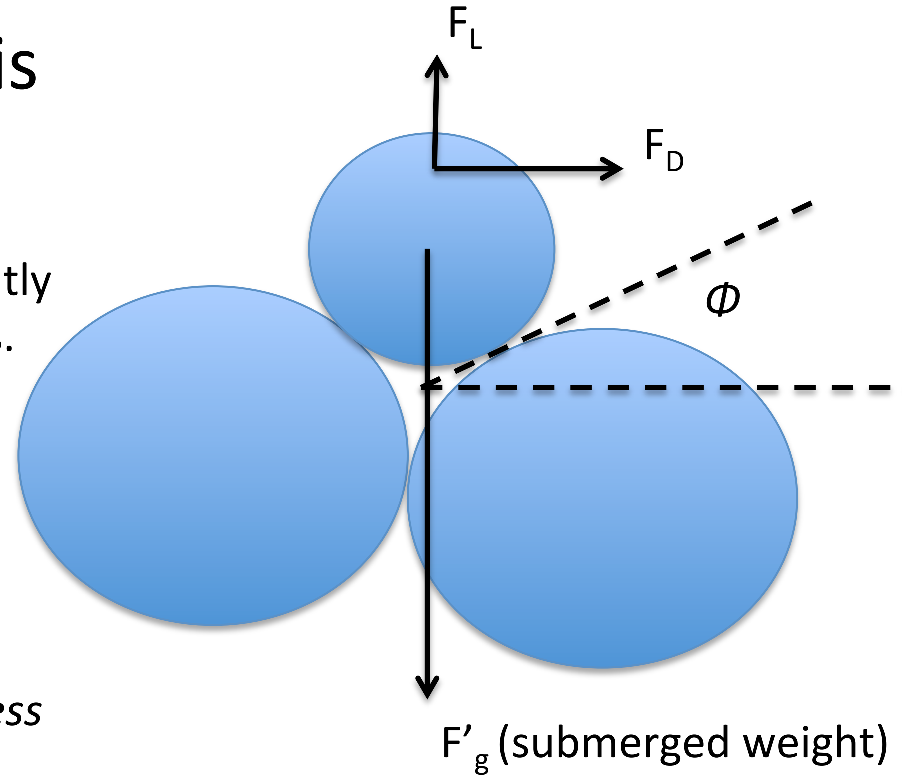
Shields number ("drag/weight ratio")

Is there a representative particle size for the bedload as a whole?
Yes: it's D_{50}.

Equal mobility hypothesis



"Hiding" effect
→ small particles
don't move significantly
before the D_{50} moves.



Trade-off between size and embeddedness

Significant controversy over validity of equal mobility hypothesis in the late '80s – early '90s.
Parameterise using

$$\tau_* = B(D/D_{50})^{\alpha}$$

$\alpha = -1$ would indicate perfect equal mobility (**no** sorting by grain size with downstream distance)
$\alpha = -0.9$ found from flume experiments (permitting long-distance sorting by grain size).

τ_{*c50} ~ 0.04, from experiments (0.045-0.047 for gravel, 0.03 for sand)

1936:

Hydraulically rough: viscous sublayer is a thin skin around the particles.

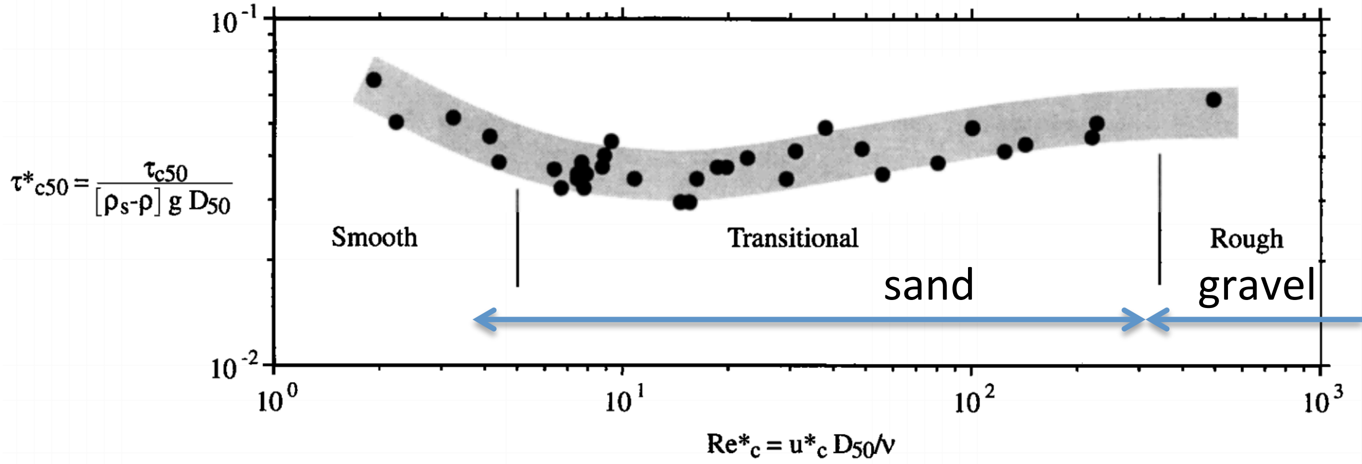
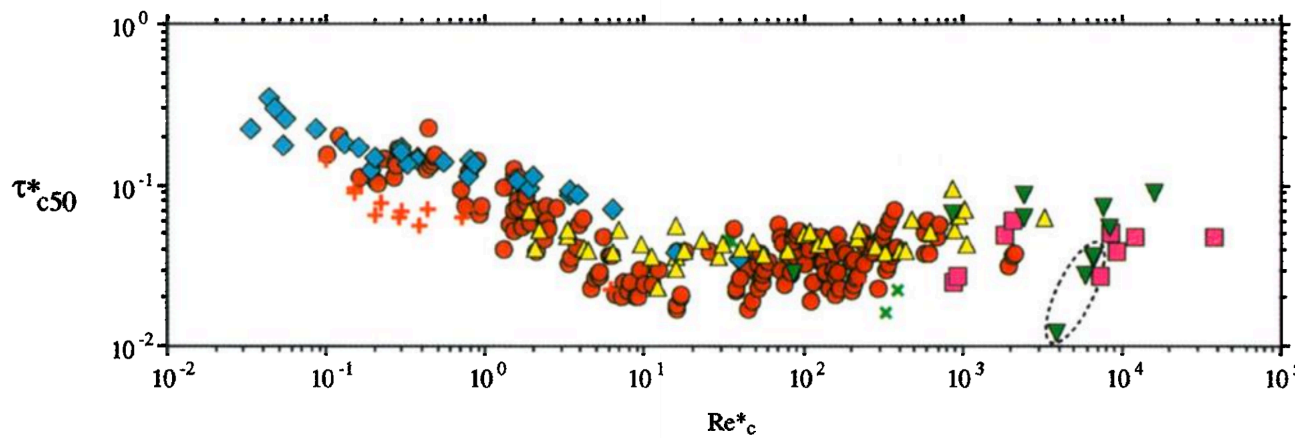


Figure 1. *Shields'* [1936] curve redrafted from *Rouse* [1939].

Re* = "Reynolds roughness number"

1999:

Theory has approximately reproduced some parts of this curve.



Causes of scatter:
(1) differing definitions of initiation of motion (most important).
(2) slope-dependence? (Lamb et al. JGR 2008)

Buffington & Montgomery, Water Resources Research, 1999

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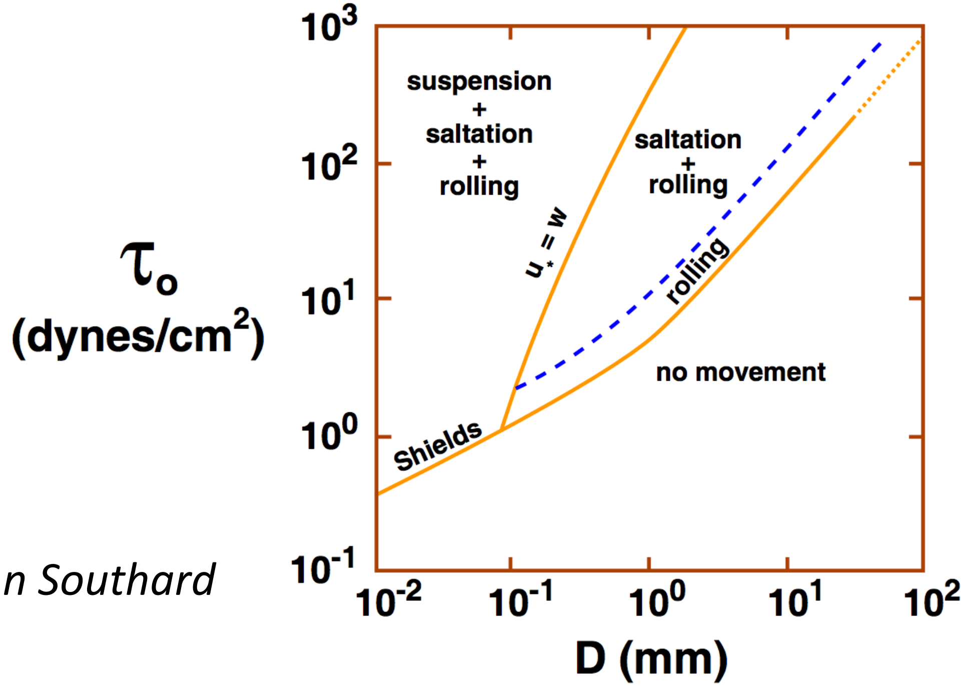
Consequences of increasing shear stress: gravel-bed vs. sand-bed rivers



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Suspension: characteristic velocity for turbulent fluctuations (u*) exceeds settling velocity (ratio is ~Rouse number).

Typical transport distance 100m/yr in gravel-bedded bedload
Sand: km/day

(Experimentally, u* is approximately equal to rms fluctuations in vertical turbulent velocity)

Empirically, rivers are either gravel-bedded or sand-bedded (little in between)
The cause is unsettled: e.g. Jerolmack & Brzinski Geology 2010 vs. Lamb & Venditti GRL 2016

Bedload transport

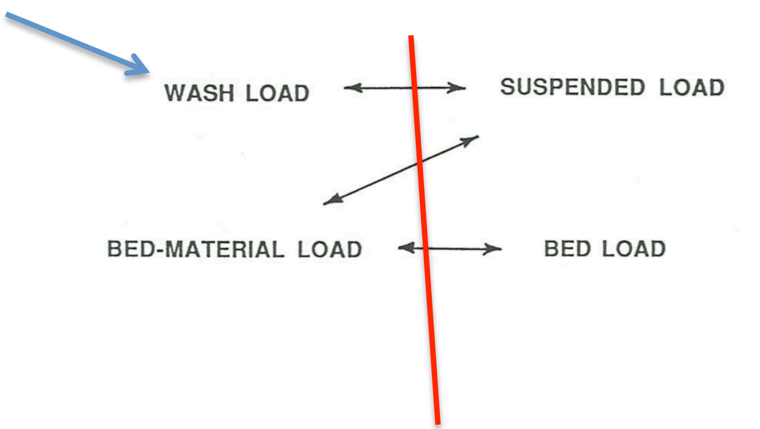
(Most common:)

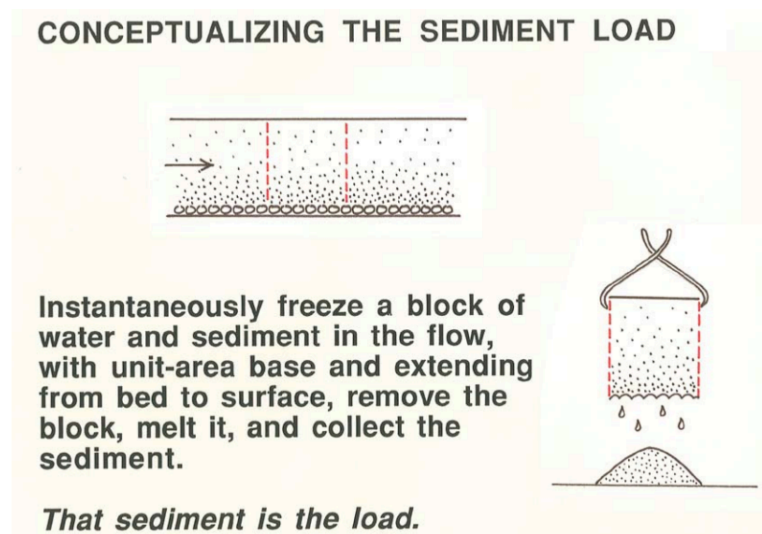
Meyer-Peter Muller

$$q_{bl} = k_b(\tau_b - \tau_c)^{3/2}$$

Many alternatives, e.g.
- Yalin
- Einstein
- Discrete element modeling

there is no theory for washload:
it is entirely controlled by upstream supply



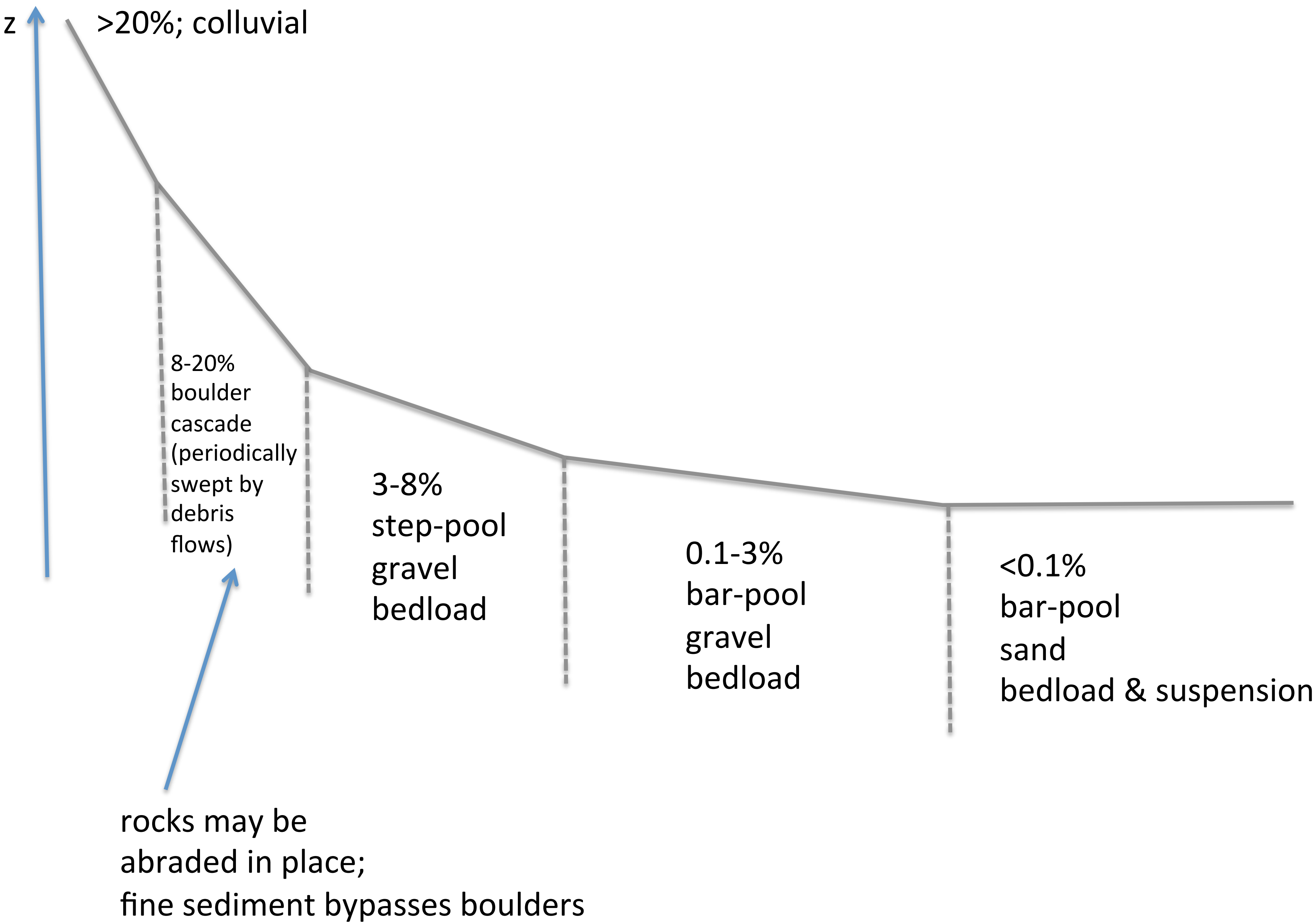


John Southard

River channel morphology and dynamics

- "Rivers are the authors of their own geometry" (L. Leopold)
  - *And of their own bed grain-size distribution.*
- Rivers have well-defined banks.
  - *Bankfull discharge 5-7 days per year; floodplains inundated every 1-2 years.*
  - *Regular geometry also applicable to canyon rivers.*
  - *Width scales as* $Q^{0.5}$
- River beds are (usually) not flat.
  - *Plane beds are uncommon. Bars and pools, spacing = 5.4x width.*
- Rivers meander.
  - *Wavelength ~ 11x channel width.*
- River profiles are concave-up.
  - *Grainsize also decreases downstream.*

Slope, grain size, and transport mechanism: strongly correlated



What sets width?

$Q = wd\langle u \rangle$

$w = aQ^b$

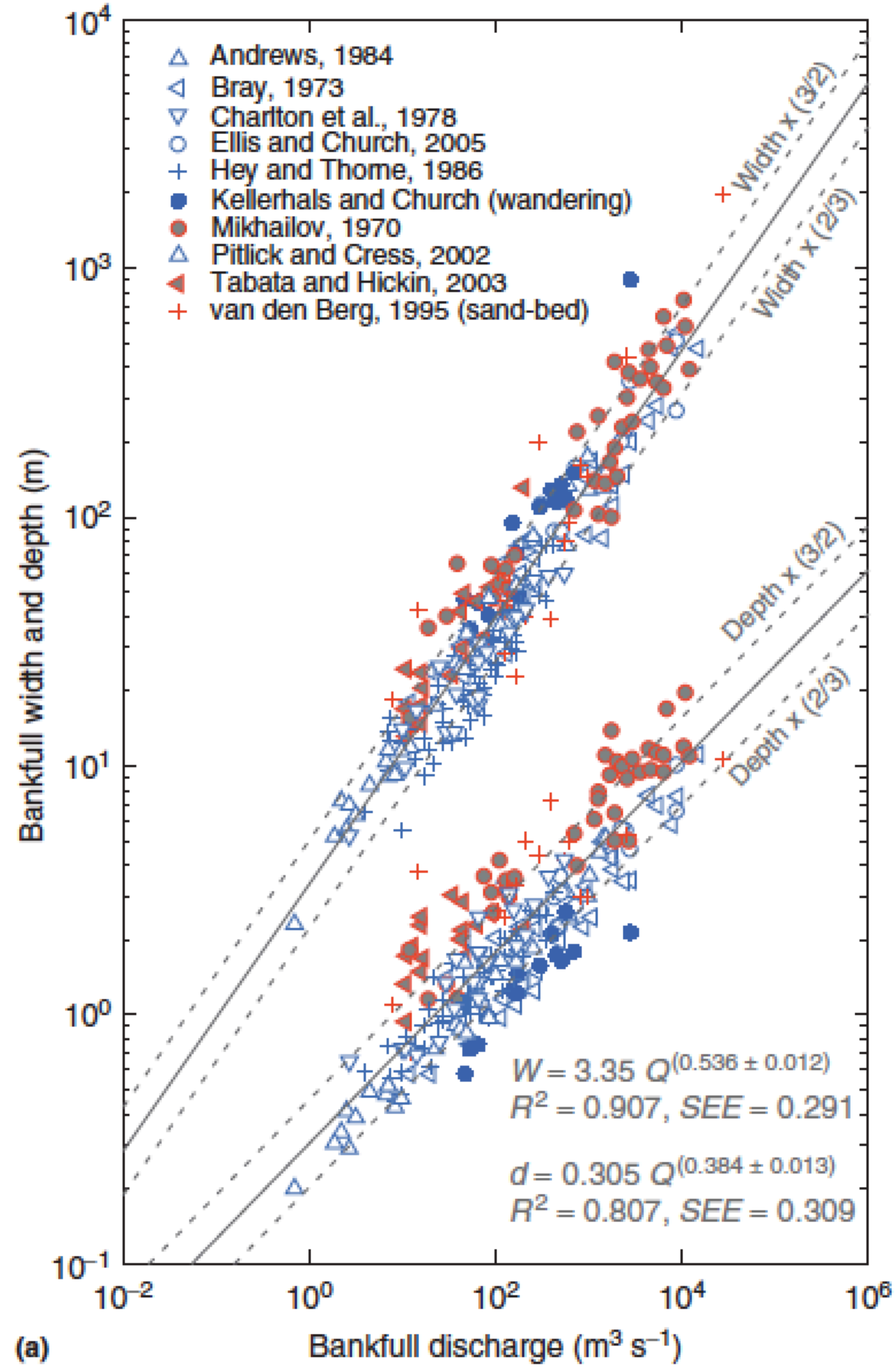
$d = cQ^f$

$\langle u \rangle = kQ^m$

$b+f+m = 1$

Comparing different points downstream → $b = 0.5$, $m = 0.1$, $f = 0.4$

Eaton, Treatise on Geomorphology, 2013



What sets width? Three approaches to this unsolved question:

(1) Posit **empirical relationships between hydraulics, sediment supply, and form** (Parker et al. 2008 in suggested reading; Ikeda et al. 1988 Water Resources Research).

(2) **Extremal hypotheses**; posit an optimum channel, minimizing energy (Examples: minimum streampower per unit length; maximum friction; maximum sediment transport rate; minimum total streampower; minimize Froude number)

(3) What is the actual mechanism? What controls what sediment does, how high the bank is, & c.?

Key points from today's lecture

- Critical Shields stress
- Differences between gravel-bed vs. sand-bed rivers
- Discharge-width scaling