

GEOS 28600

***The science of landscapes:***

# **Earth & Planetary Surface Processes**

[http://geosci.uchicago.edu/~kite/geos28600\\_2019/](http://geosci.uchicago.edu/~kite/geos28600_2019/)

Lecture 9

Wednesday 13 Feb 2019

Continuation of fluvial sediment transport

→ What controls the shape of rivers?

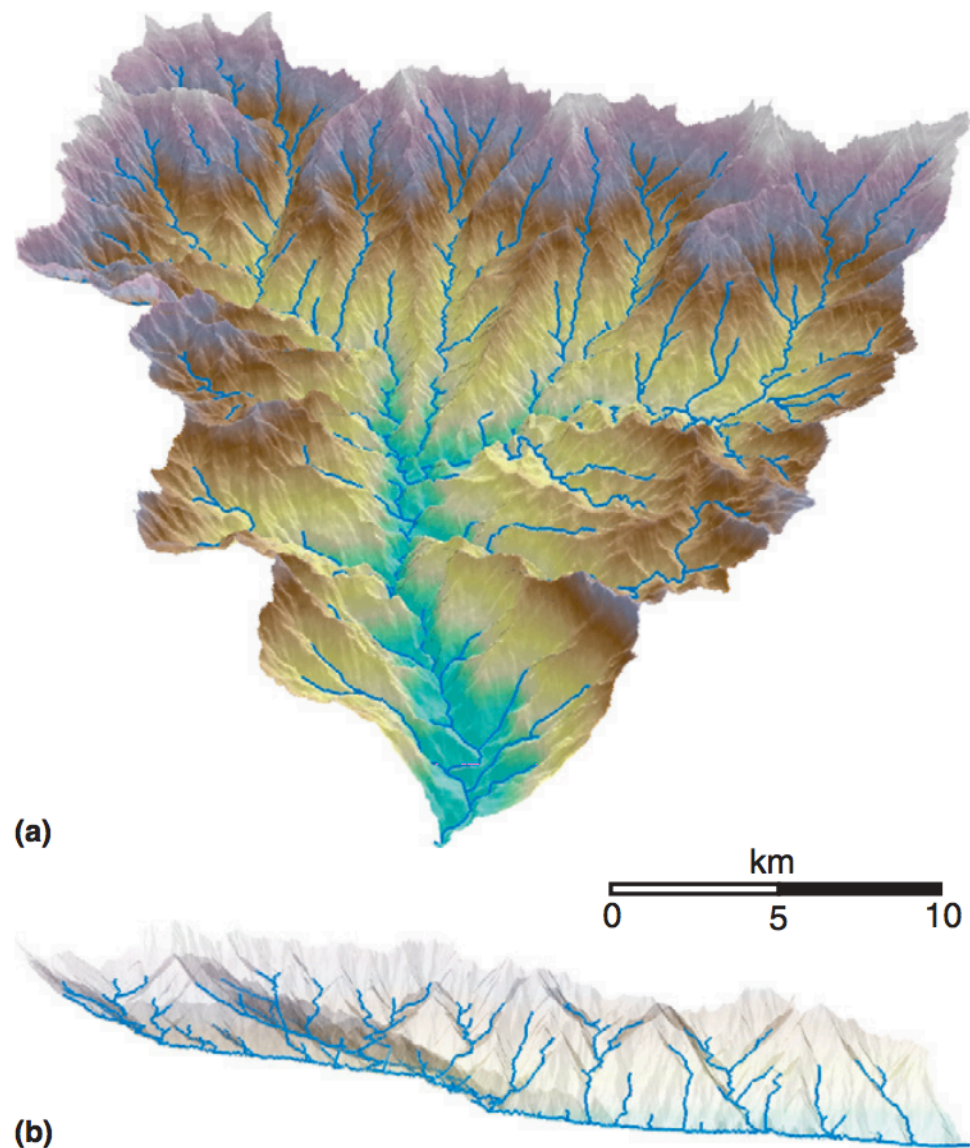
# Logistics

- Class on Friday 15 (& Fri 22): HGS 180, 9:30a

Why care about fluvial sediment transport?

**Earth:** most of Earth's relief is controlled by bedrock river processes (which depend on the sediment supply from hillslopes).

Bedrock rivers are the skeleton of Earth's landscape



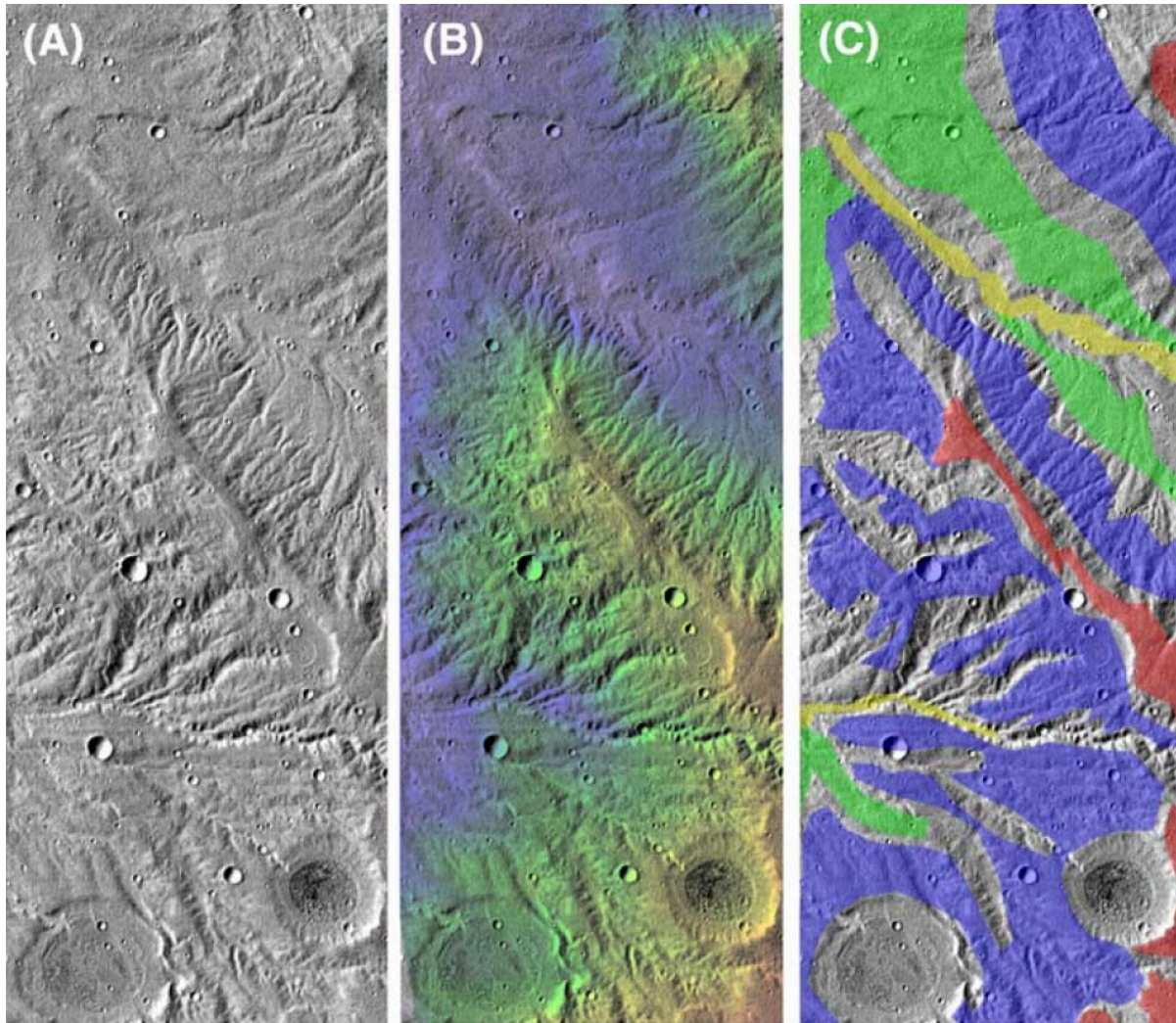
**Figure 1** (a) Perspective view of a steep mountain catchment in Taiwan (Liwu catchment, 535 km<sup>2</sup> drainage area, 3650 m relief). Channel segments with upstream drainage area greater than 0.8 km<sup>2</sup> are highlighted in blue. (b) Side-view of this catchment with the topography made transparent to highlight the relation between local relief and the elevation drop on bedrock channels note the knickpoint

**Earth** is the only planet on which fluvial erosion is known to dominate –  $8 \text{ km}^3/\text{yr}$  (pre-dam). **Mars'** valley networks represent  $\sim 2 \text{ m}$  global equivalent layer of erosion –  $10^5 \text{ yr}$  of Earth erosion. Most of the ideas from the next two lectures assume a dominant role for fluvial(+/- hillslope) processes, which is not true\* for Mars' obvious-from-orbit  $10^3 \text{ km}$ -scale valley networks.

\*Aharonson et al. PNAS 2002

**Titan** topography (which is gappy!) suggests fluvial erosion is important, at least regionally.

Why care about fluvial sediment transport?



Howard et al. JGR 2005



# Fluvial sediment transport

**BEDLOAD, RIVER GEOMETRY**

# Key points from “Introduction to fluvial sediment transport”:

- “Law of the wall” – how to calculate river discharge from elementary measurements (bed grain size and river depth).
- Critical Shields stress
- Differences between gravel-bed vs. sand-bed rivers
- Discharge-width scaling

## Drag coefficient for bed particles:

$$\rightarrow \tau_b = \rho g R S = C_D \rho \langle u \rangle^2 / 2$$

$$\langle u \rangle = (2g R S / C_D)^{1/2}$$

$$(2g / C_D)^{1/2} = C = \text{Chezy coefficient}$$

$$\langle u \rangle = C (R S)^{1/2}$$

Chezy equation (1769)

$$\langle u \rangle = (8g / f)^{1/2} (R S)^{1/2}$$

$f$  = Darcy-Weisbach friction factor

$$\langle u \rangle = R^{2/3} S^{1/2} n^{-1}$$

$n$  = Manning roughness coefficient

Most used, because lots of investment in measuring  $n$  for different objects

$0.025 < n < 0.03$  ----- Clean, straight rivers (no debris or wood in channel)

$0.033 < n < 0.03$  ----- Winding rivers with pools and riffles

$0.075 < n < 0.15$  ----- Weedy, winding and overgrown rivers

$n = 0.031(D_{84})^{1/6}$  ----- Straight, gravelled rivers

In sand-bedded rivers (e.g. Mississippi), form drag due to sand dunes is important.

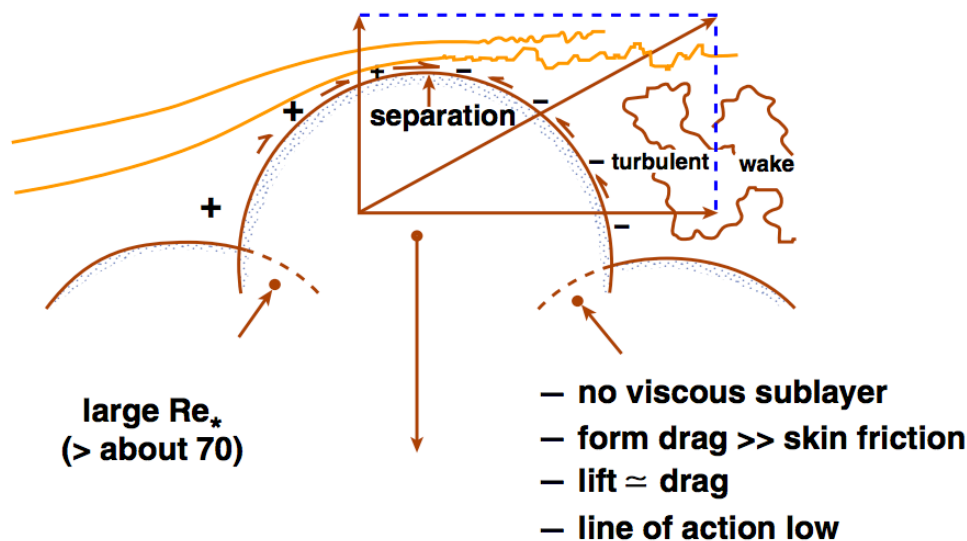
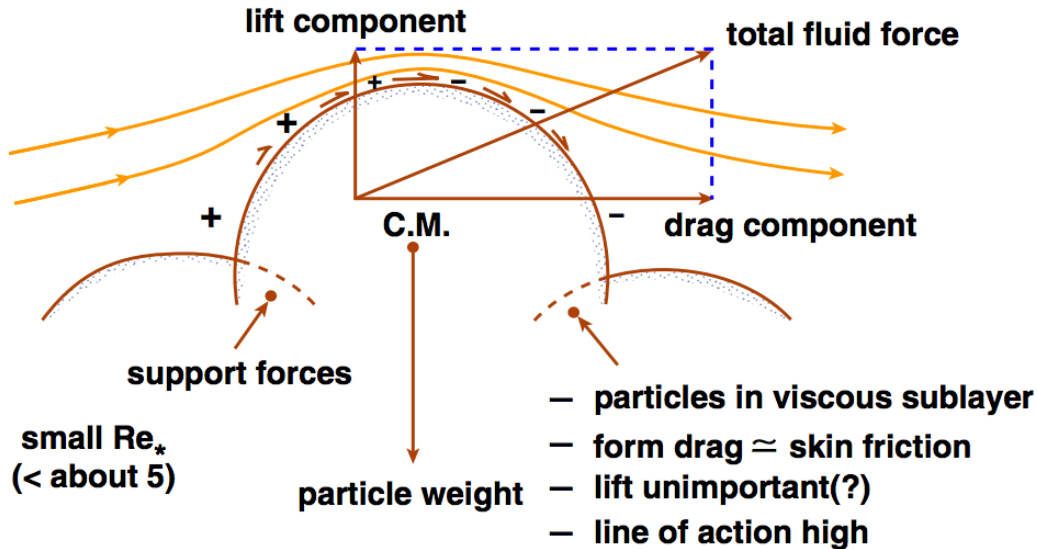
In very steep streams, supercritical flow may occur:

*supercritical flow*

Froude number

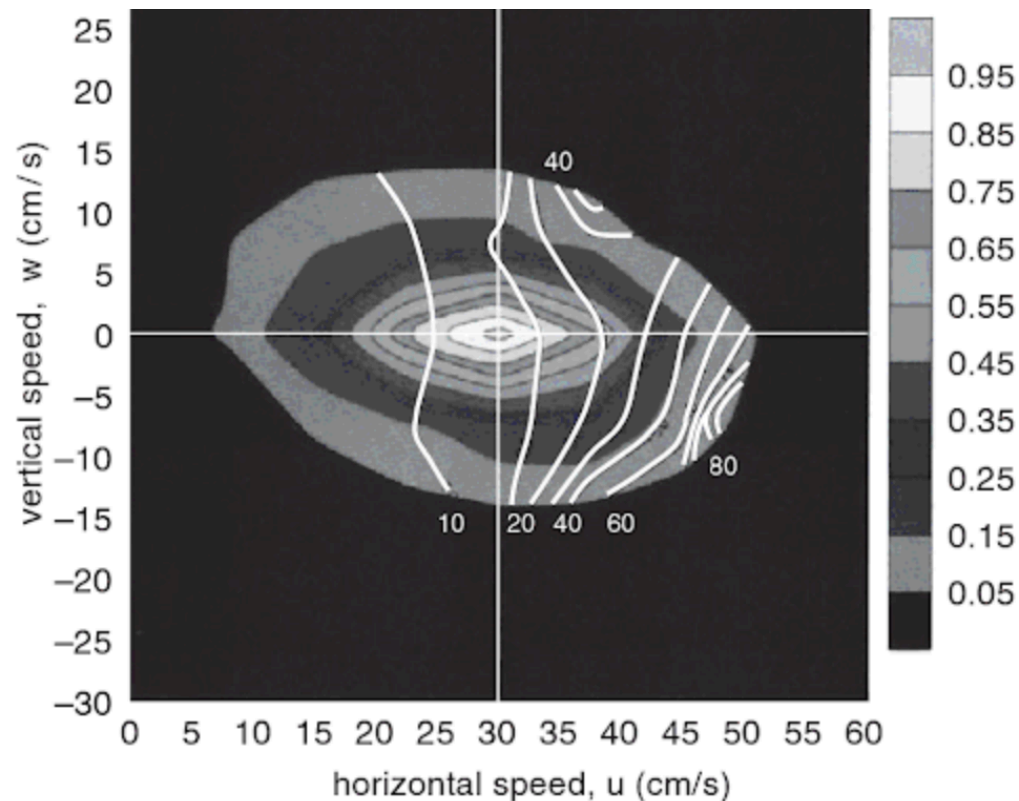
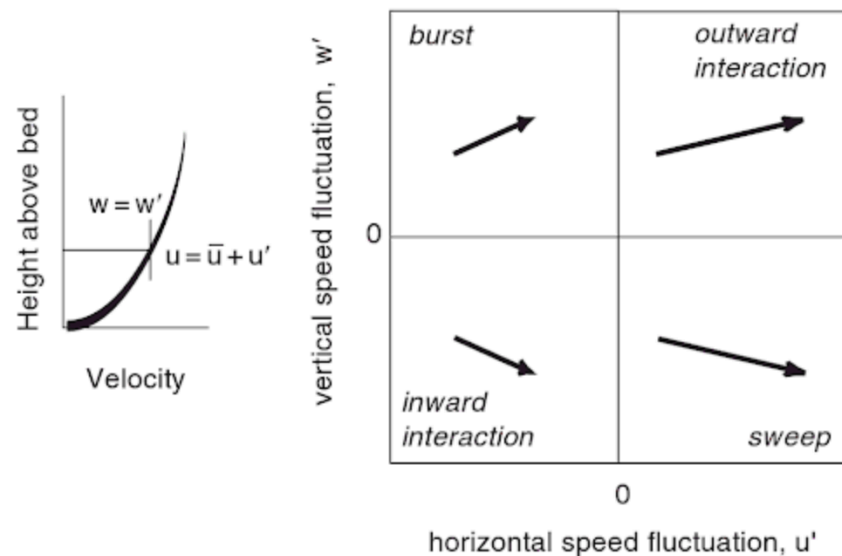
$$Fr \# = \langle u \rangle / (gh)^{1/2} > 1$$

# Getting from water flow to sediment flux



*John Southard*





**Figure 14.7** Sediment transport rates (contoured in grains/cm/s) and joint probabilities (scale on right) of streamwise ( $u$ ) and vertical ( $w$ ) velocities averaged over several experimental runs. Mean horizontal flow speed is 30 cm/s, while mean vertical speed is zero. Note that the greatest transport rates correspond to the lower right quadrant, in which turbulence brings high horizontal speed fluid downward toward the bed (after Nelson

from Anderson & Anderson  
'Geomorphology' text

# Sediment transport in rivers: (Shields number)

At the initiation of grain motion,

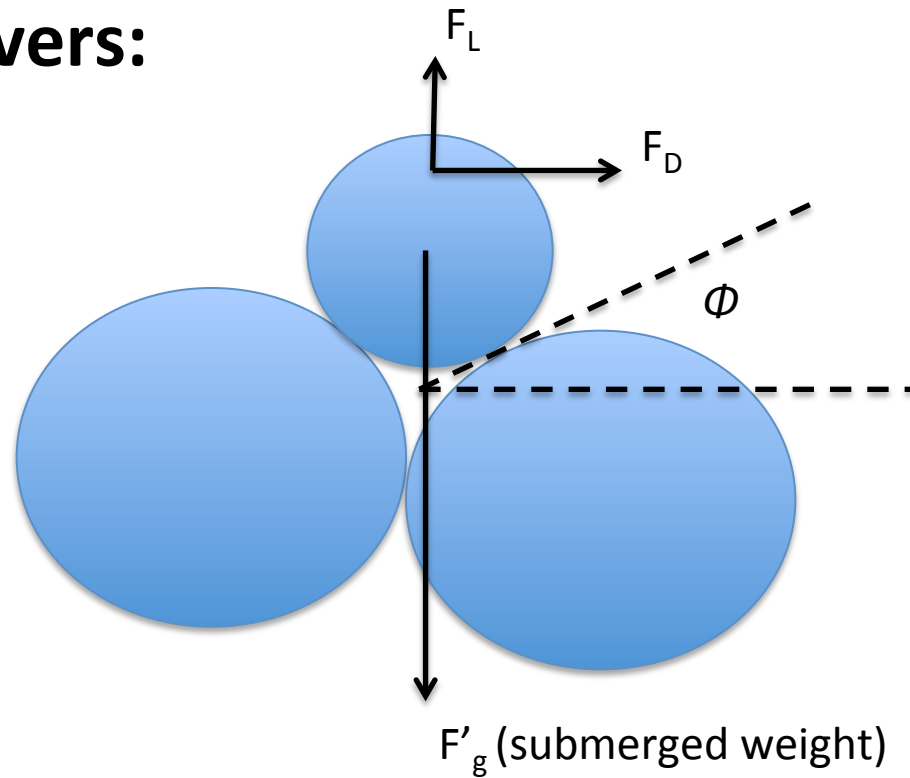
$$F_D = (F'_g - F_L) \tan \phi$$

$$\rightarrow F_D / F'_g = \frac{\tan \phi}{1 + (F_L / F_D) \tan \phi}$$

$$\approx \frac{\tau_c D^2}{(\rho_s - \rho) g D^3}$$

$$= \frac{\tau_c}{(\rho_s - \rho) g D} = \tau_*$$

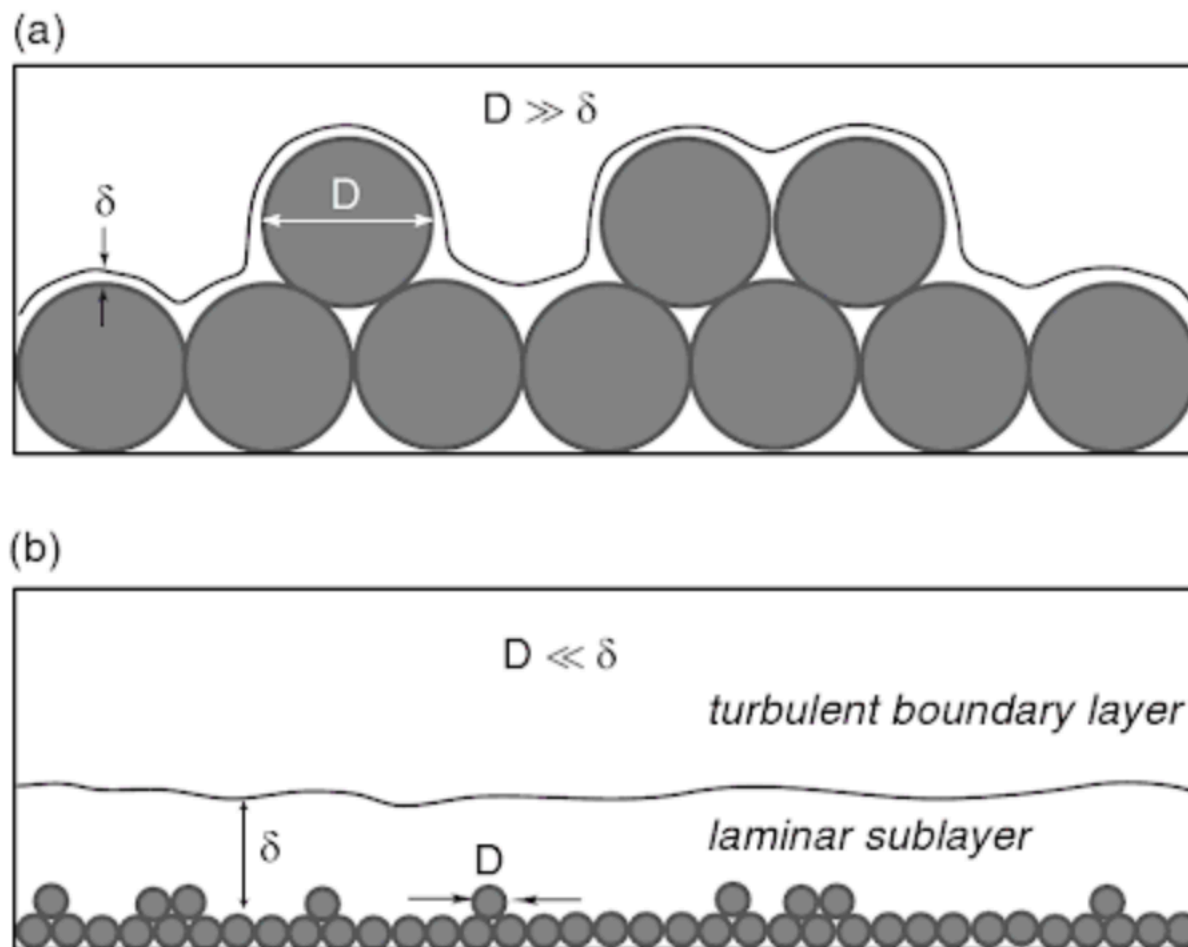
Shields number (“drag/weight ratio”)



Is there a representative particle size for the bedload as a whole?

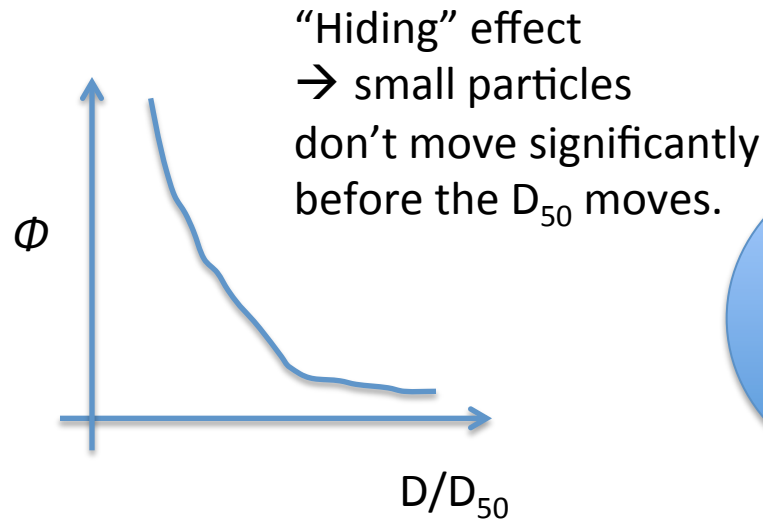
Yes: it's  $D_{50}$ .

## Thought experiment: which grains are hardest to entrain?

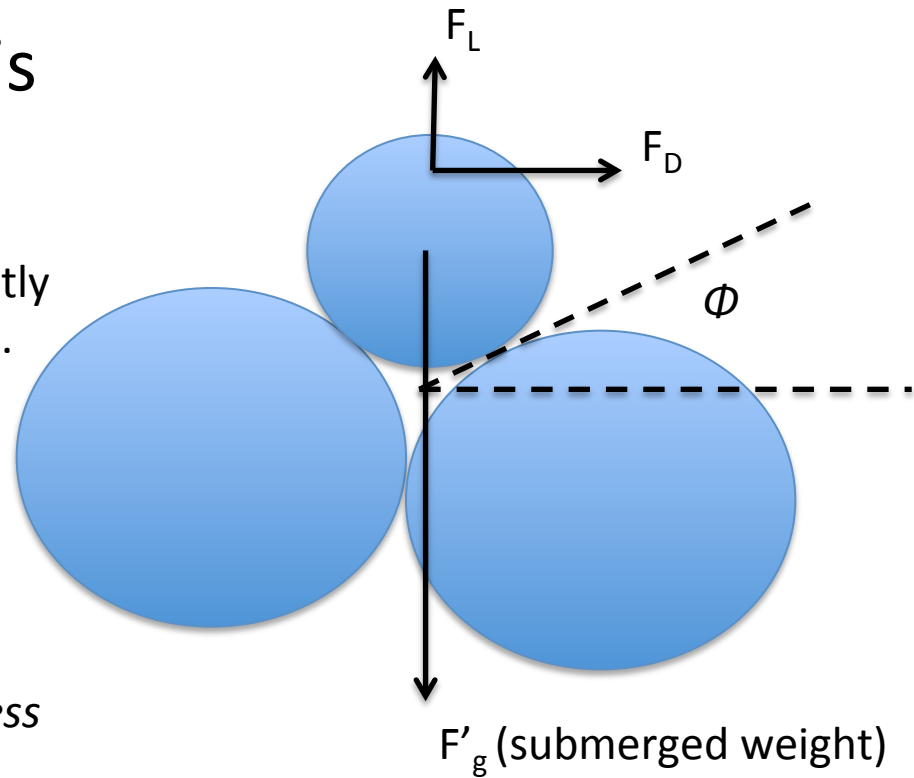


**Figure 14.4** Grains on the bed beneath a turbulent flow. Grains whose diameter,  $D$ , is smaller than the laminar sub-layer are effectively shielded from turbulence, and will be more difficult to entrain.

# Equal mobility hypothesis



*Trade-off between size and embeddedness*



Significant controversy over validity of equal mobility hypothesis in the late '80s – early '90s.  
Parameterise using

$$\tau_* = B(D/D_{50})^\alpha$$

$\alpha = -1$  would indicate perfect equal mobility (**no** sorting by grain size with downstream distance)  
 $\alpha = -0.9$  found from flume experiments (permitting long-distance sorting by grain size).



$\tau_{*c50} \sim 0.04$ , from experiments (0.045-0.047 for gravel, 0.03 for sand)

1936:

$$\tau_{*c50} = \frac{\tau_{c50}}{[\rho_s - \rho] g D_{50}}$$

Hydraulically rough:  
viscous sublayer is a thin  
skin around the particles.

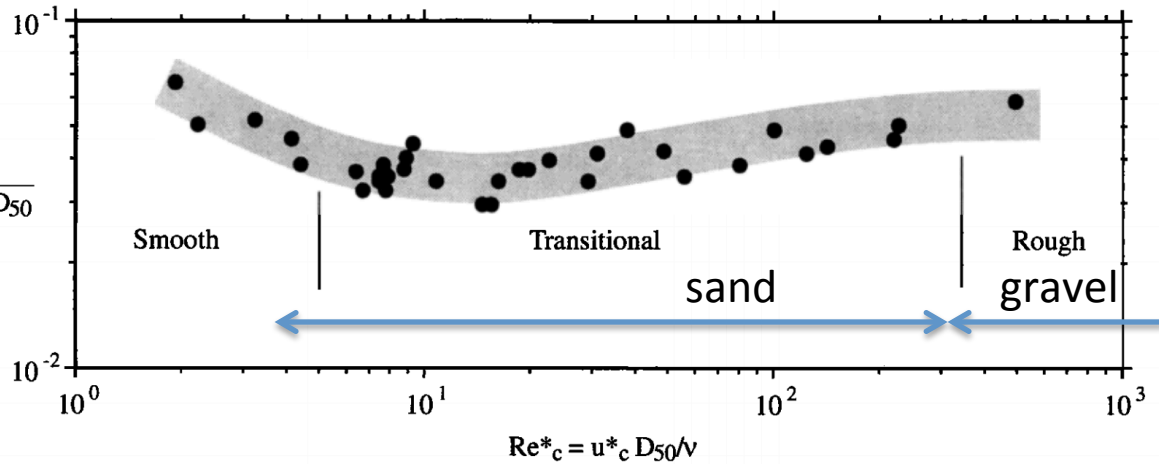
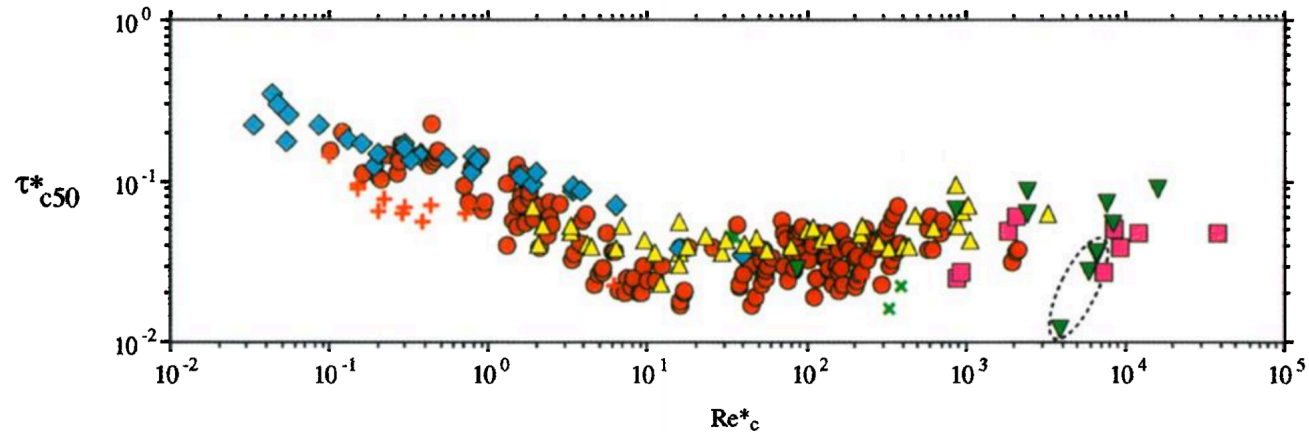


Figure 1. Shields' [1936] curve redrafted from Rouse [1939].

$Re^*$  = "Reynolds roughness number"

1999:

Theory has approximately  
reproduced some parts  
of this curve.



Causes of scatter:  
(1) differing definitions of  
initiation of motion (most important).  
(2) slope-dependence?  
(Lamb et al. JGR 2008)

Buffington & Montgomery, Water Resources Research, 1999

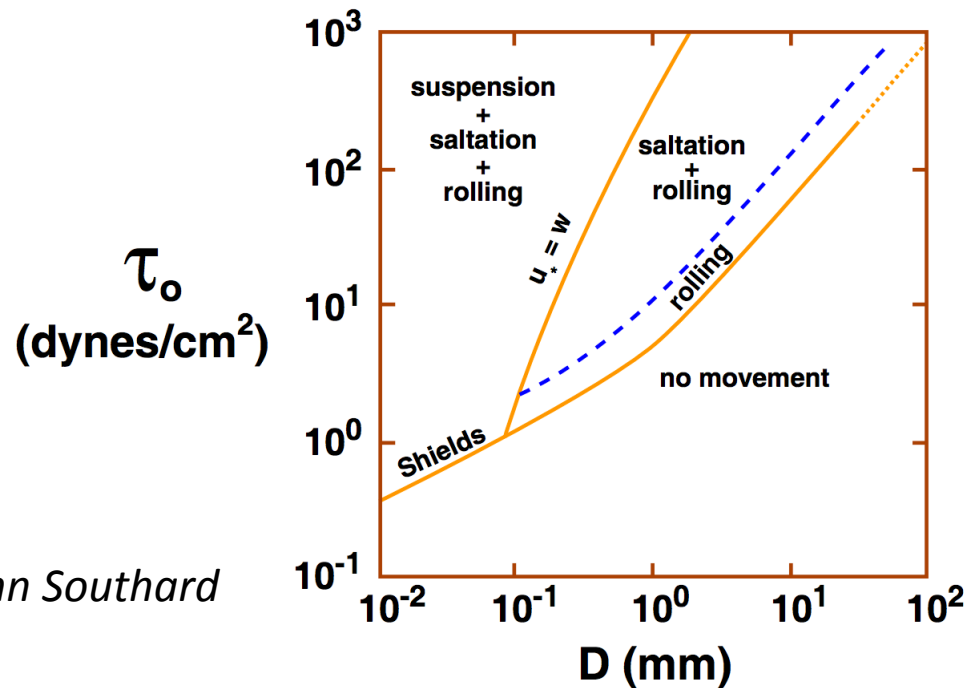
# Fluvial sediment transport: introduction

REVIEW OF REQUIRED READING (SCHOOF & HEWITT 2013)

TURBULENT VELOCITY PROFILES, INITIATION OF  
MOTION

**BEDLOAD, RIVER GEOMETRY**

# Consequences of increasing shear stress: gravel-bed vs. sand-bed rivers



John Southard

Suspension: characteristic velocity for turbulent fluctuations ( $u^*$ ) exceeds settling velocity (ratio is  $\sim$ Rouse number).

Typical transport distance  
100m/yr in gravel-bedded bedload  
Sand: km/day

(Experimentally,  $u^*$  is approximately equal to rms fluctuations in vertical turbulent velocity)

Empirically, rivers are either gravel-bedded or sand-bedded (little in between)  
The cause is unsettled: e.g. Jerolmack & Brzinski Geology 2010 vs. Lamb & Venditti GRL 2016

# Bedload transport

(Most common:)

Meyer-Peter Muller

$$q_{bl} = k_b (\tau_b - \tau_c)^{3/2}$$

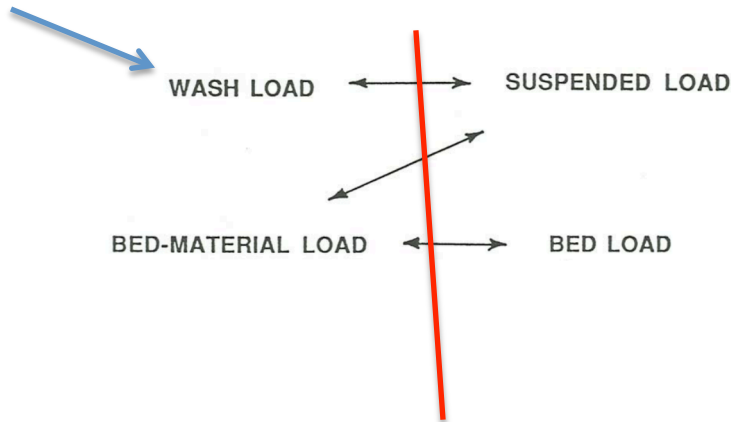
Many alternatives, e.g.

Yalin

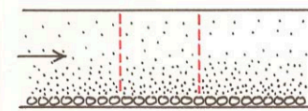
Einstein

Discrete element modeling

*there is no theory for washload:  
it is entirely controlled by upstream supply*

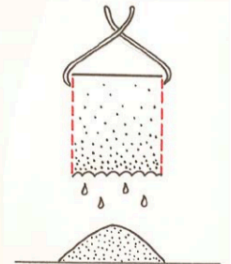


## CONCEPTUALIZING THE SEDIMENT LOAD



Instantaneously freeze a block of water and sediment in the flow, with unit-area base and extending from bed to surface, remove the block, melt it, and collect the sediment.

*That sediment is the load.*



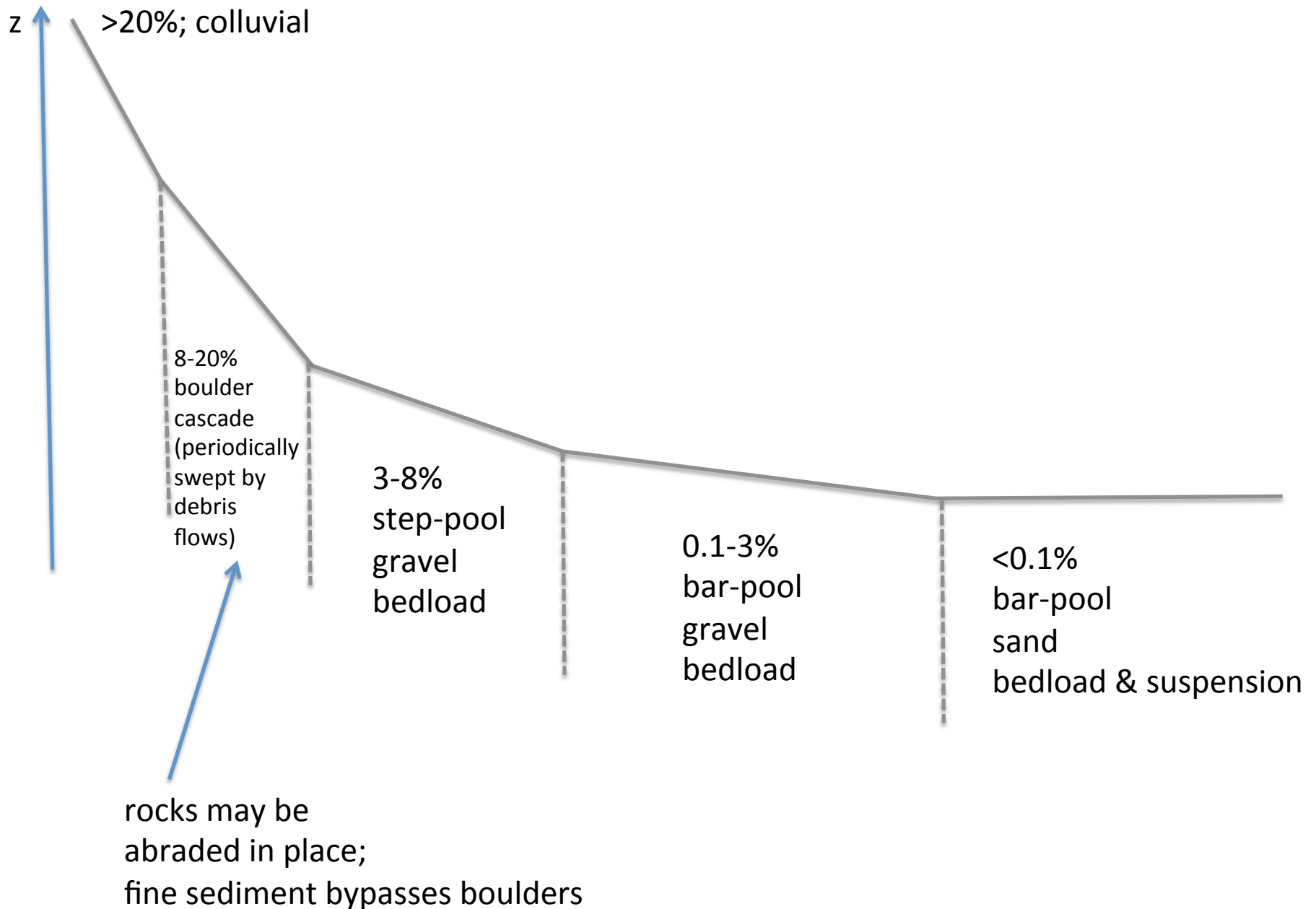
John Southard



# River channel morphology and dynamics

- “Rivers are the authors of their own geometry” (L. Leopold)
  - *And of their own bed grain-size distribution.*
- Rivers have well-defined banks.
  - *Bankfull discharge 5-7 days per year; floodplains inundated every 1-2 years.*
  - *Regular geometry also applicable to canyon rivers.*
  - *Width scales as  $Q^{0.5}$*
- River beds are (usually) not flat.
  - *Plane beds are uncommon. Bars and pools, spacing = 5.4x width.*
- Rivers meander.
  - *Wavelength  $\sim 11$ x channel width.*
- River profiles are concave-up.
  - *Grainsize also decreases downstream.*

# Slope, grain size, and transport mechanism: strongly correlated



# What sets width?

$$Q = wd\langle u \rangle$$

$$w = aQ^b$$

$$d = cQ^f$$

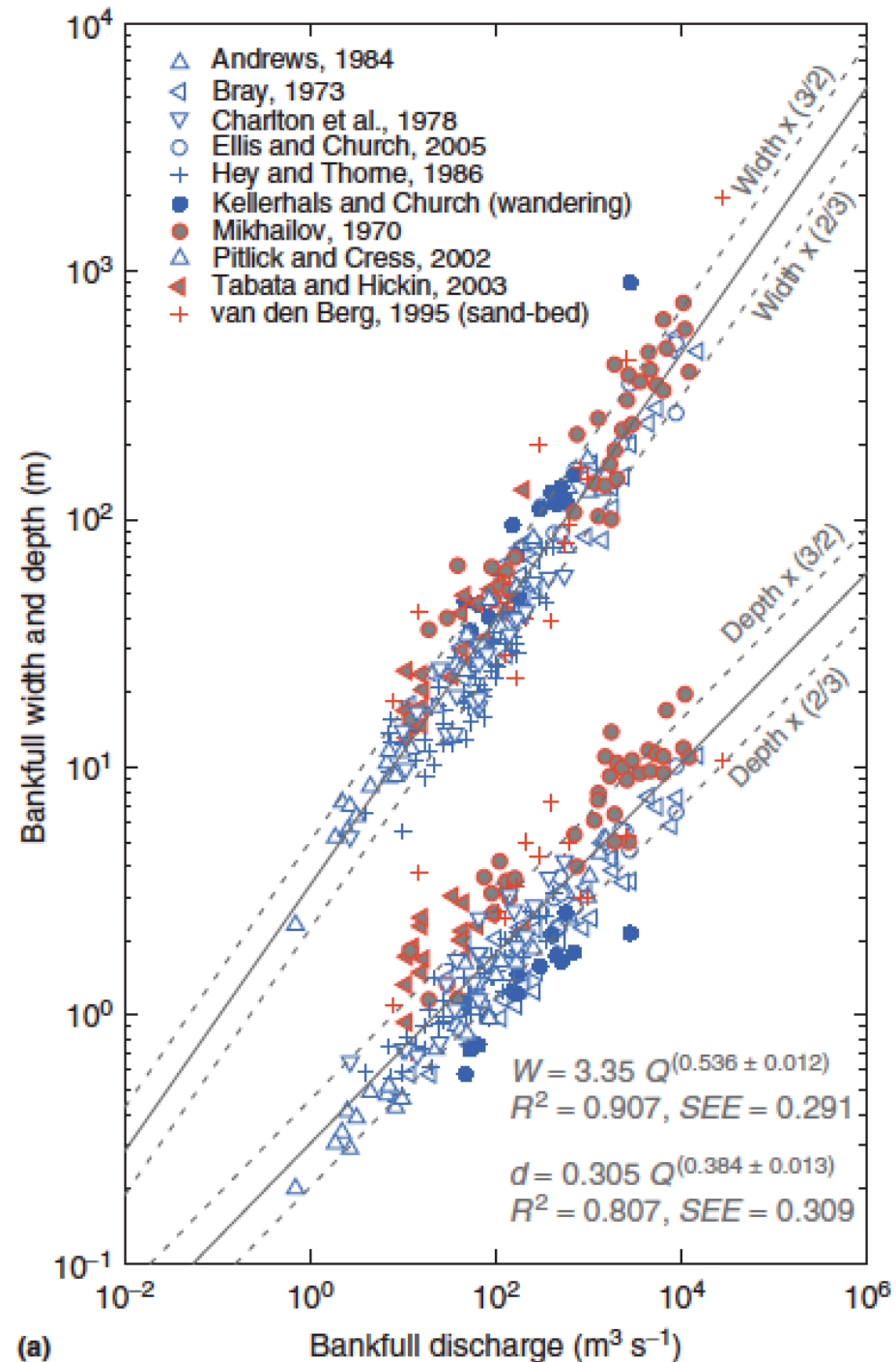
$$\langle u \rangle = kQ^m$$

$$b+f+m = 1$$

Comparing  
different points  
downstream

$b = 0.5$   
 $m = 0.1$   
 $f = 0.4$

Eaton, Treatise on  
Geomorphology, 2013



What sets width? Three approaches to this unsolved question:

- (1) Posit **empirical relationships between hydraulics, sediment supply, and form** (Parker et al. 2008 in suggested reading; Ikeda et al. 1988 Water Resources Research).
- (2) **Extremal hypotheses**; posit an optimum channel, minimizing energy (Examples: minimum streampower per unit length; maximum friction; maximum sediment transport rate; minimum total streampower; minimize Froude number)
- (3) What is the actual mechanism? What controls what sediment does, how high the bank is, & c.?



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- Law of the wall – how to calculate river discharge from elementary measurements (bed grain size and river depth).
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# Rivers and landscape evolution

# Bedrock rivers

Rivers that cut into bedrock (“rock is everywhere close to the surface and may be frequently exposed during flood events or on decadal to centennial timescales”, Whipple et al. 2013)

**Detachment-limited landscape evolution ( $dz/dt \sim k dz/dx$ )**

Clasts transported mainly by bedload

Knickpoints can propagate  
gm

What controls the rate of  
downcutting?

What controls the height/  
width/shape of mountain belts on  
Earth? *Neil Humphrey, UWYO*





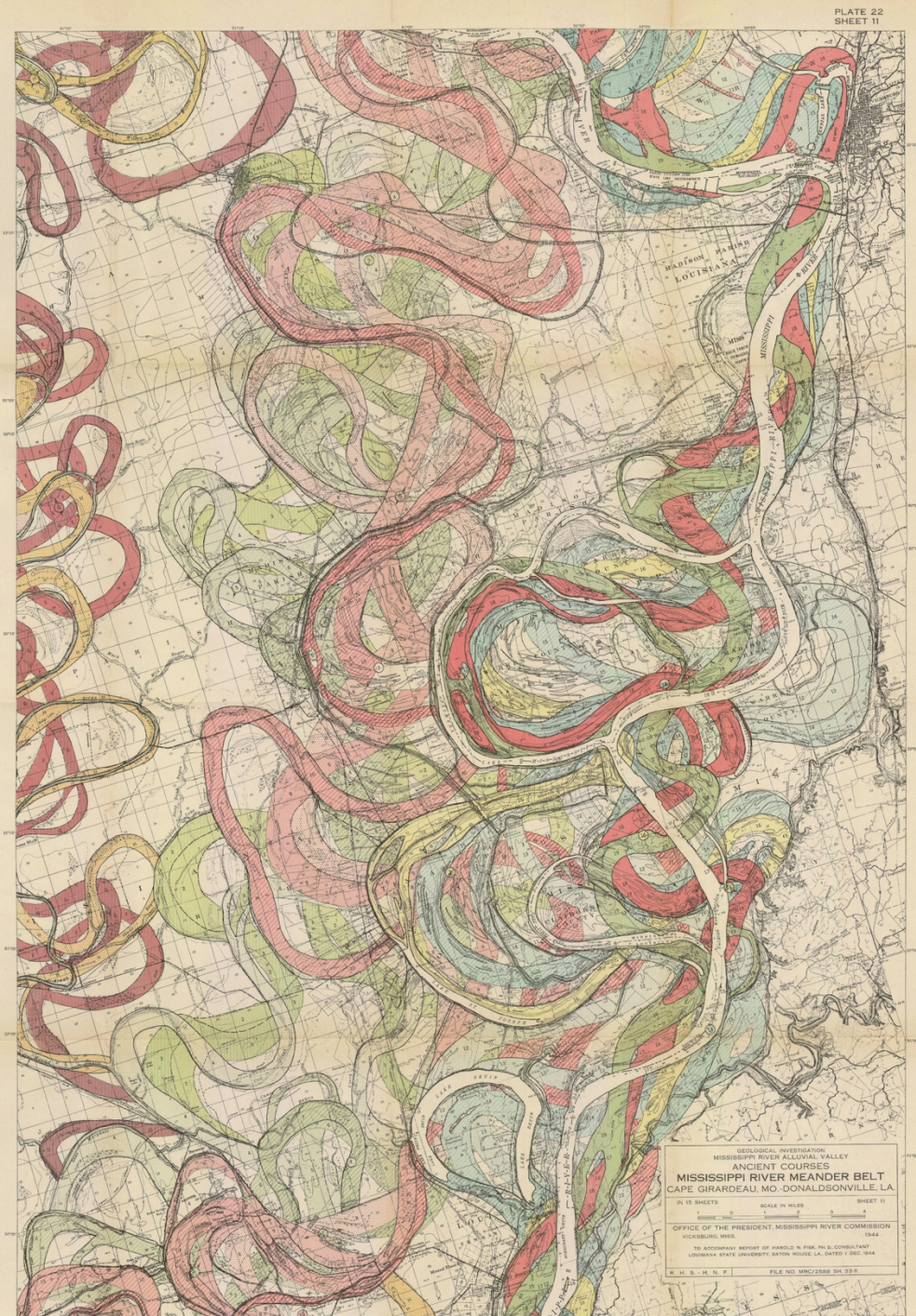
# Alluvial rivers

Transport-limited  
landscape evolution  
( $dz/dt \sim k d^2z/dx^2$ )

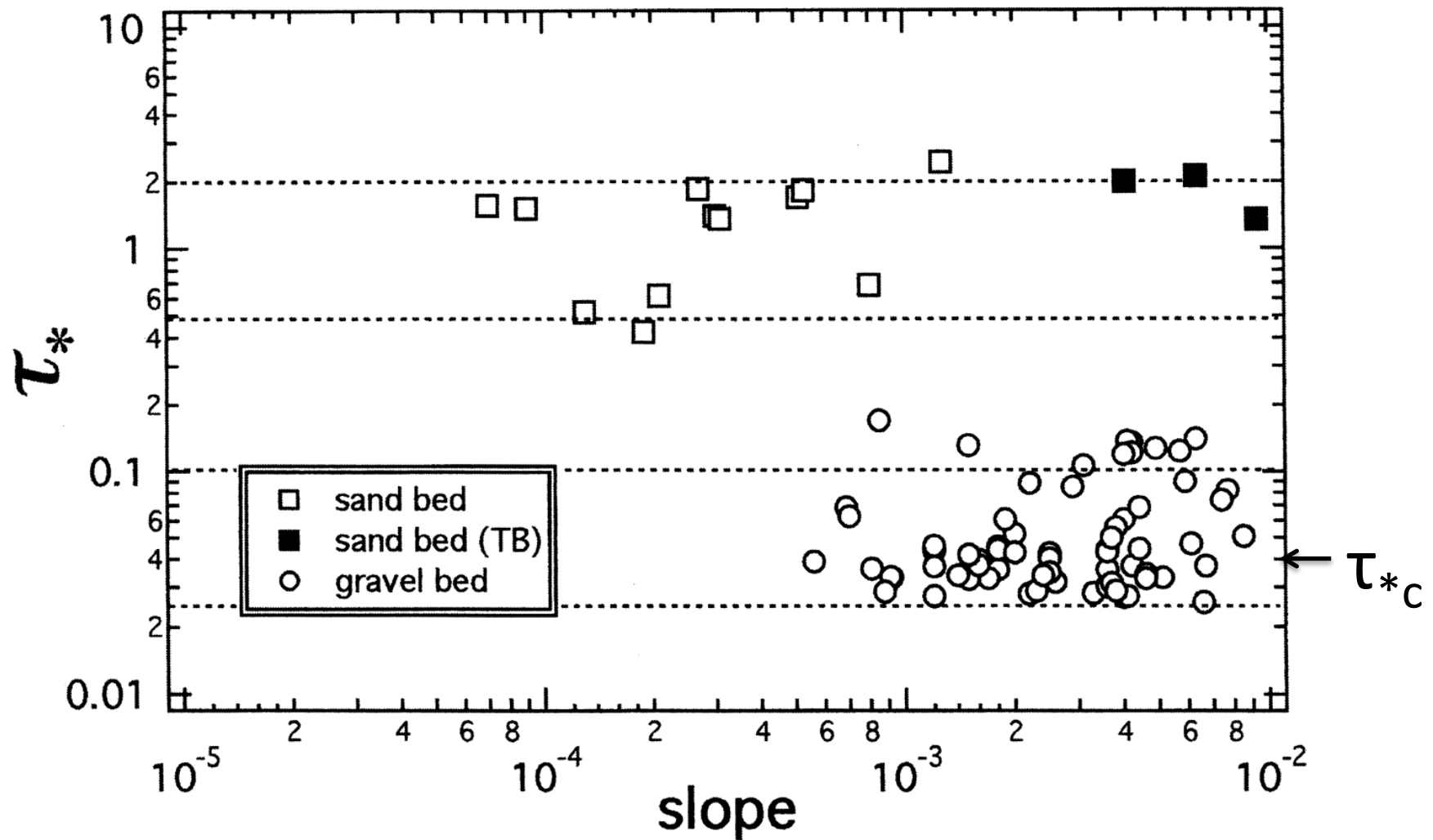
A large percentage of  
clast transport in  
suspension

When does  
meandering  
occur?

US Army Corps of Engineers  
(Lower Mississippi / Fisk 1944)



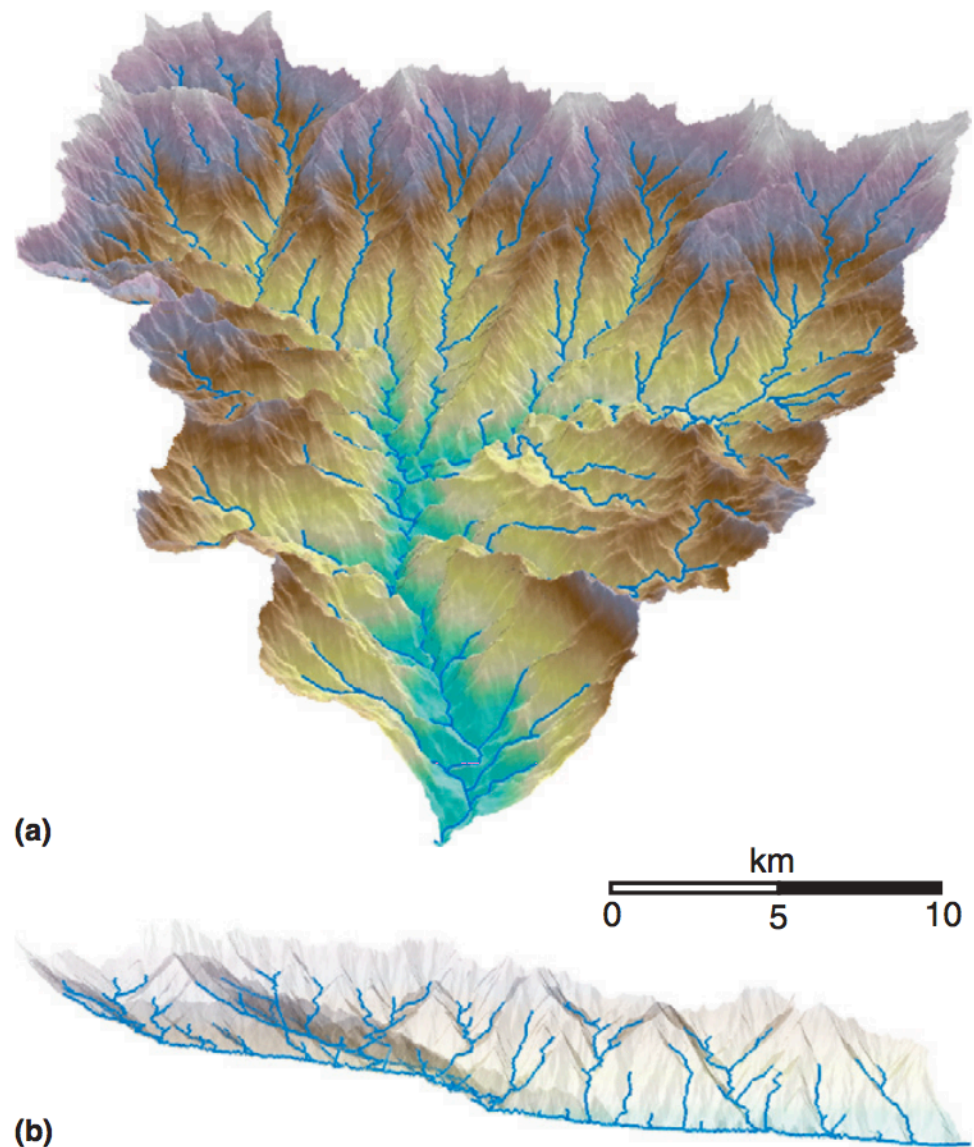
Usually (not always), Bedrock rivers  $\leftrightarrow$  gravel bed.  
Alluvial rivers can be sand or gravel bed.





**Earth:** most of Earth's relief is controlled by bedrock river processes (which depend on the sediment supply from hillslopes).

Bedrock rivers are the skeleton of Earth's landscape



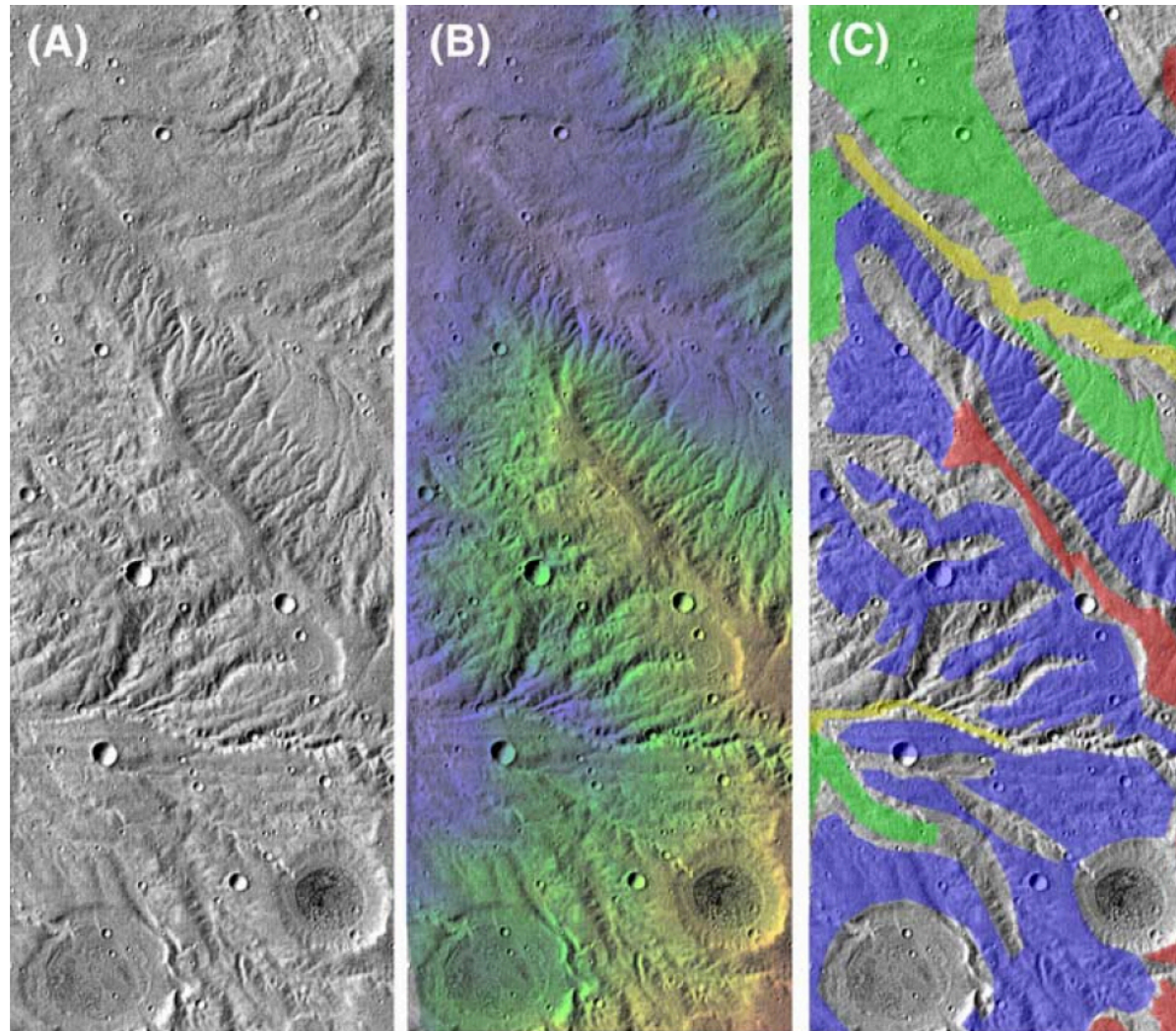
**Figure 1** (a) Perspective view of a steep mountain catchment in Taiwan (Liwu catchment, 535 km<sup>2</sup> drainage area, 3650 m relief). Channel segments with upstream drainage area greater than 0.8 km<sup>2</sup> are highlighted in blue. (b) Side-view of this catchment with the topography made transparent to highlight the relation between local relief and the elevation drop on bedrock channels note the knickpoint

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\*Aharonson et al. PNAS 2002

**Titan** topography (which is gappy!) suggests fluvial erosion is important, at least regionally.



Howard et al. JGR 2005

# Key points from today's lecture

- Advective vs. diffusive channel profile evolution
- Hypotheses for controls on concavity
- Know the streampower equation
- Know bedrock erosion processes

*In the required reading, don't omit the section on transient evolution (bridge to next lecture)*

*Next lecture: Landscape-scale responses to forcing*



Fluvial sediment transport: what controls the shape of rivers?

BASIC EQUATIONS GOVERNING BED ELEVATION

WHAT SETS RIVER PLANFORM?

WHAT SETS RIVER LONG PROFILE?

BEDROCK RIVER EROSION

Fluvial sediment transport: what controls the shape of rivers?

**EXNER EQUATION**

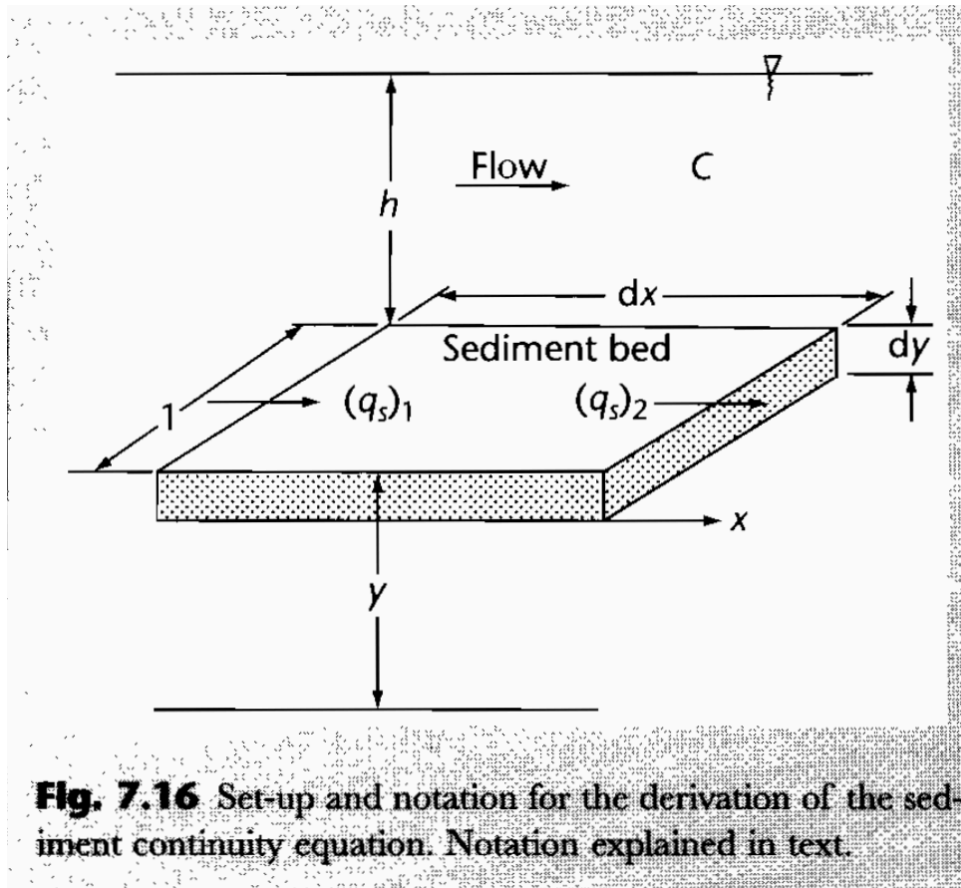
WHAT SETS RIVER PLANFORM?

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BEDROCK RIVER EROSION

# Mass balance for alluvial rivers (Exner equation)

Emphasizes suspended sediment



$C$  = sediment concentration per unit volume

$q_s$  = sediment flux

$\lambda$  = porosity in bed material

$$dh \, dx = -\frac{1}{1-\lambda} dq_s \, dt$$

$$dh \, dx = -\frac{1}{1-\lambda} (dC \cdot h) dx$$

$$\frac{dh}{dt} = -\frac{1}{1-\lambda} \left( \frac{dq_s}{dx} + h \frac{dC}{dt} \right)$$

Steady sediment supply:

$$\sigma(x) + \frac{dh}{dt} = -\frac{1}{1-\lambda} \frac{dq_s}{dx}$$

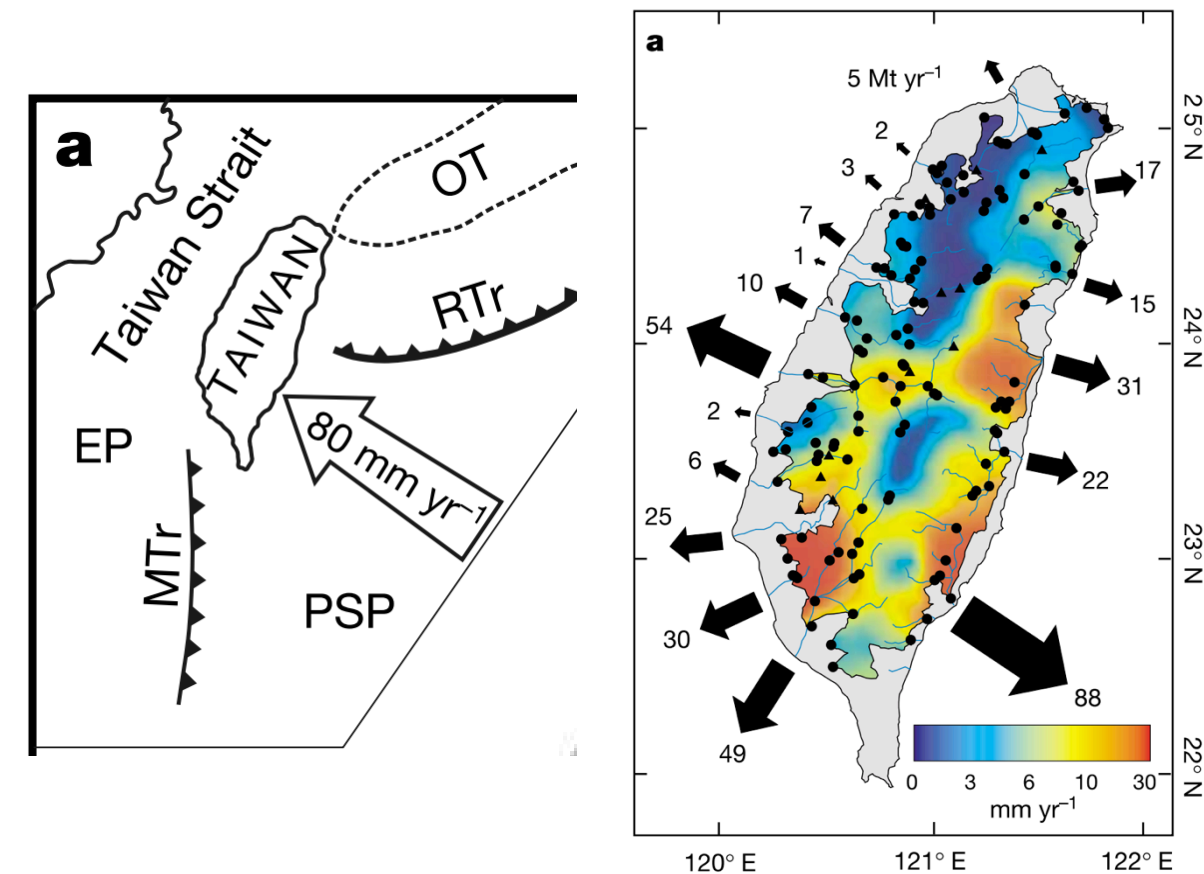
**Usually, gross transport  $\gg$  net accumulation**

# Natural laboratory: Taiwan

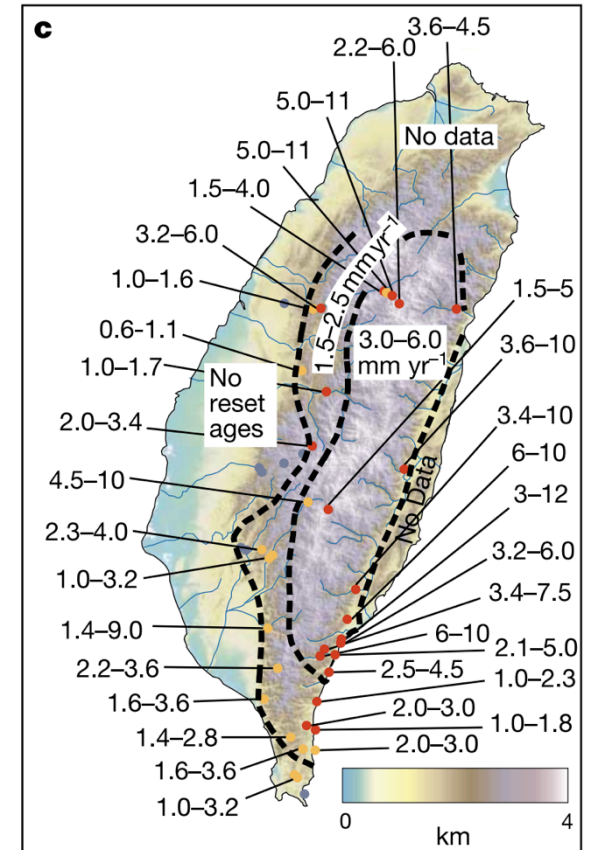
High mass fluxes forced by tectonics (uplift 2-5 mm/yr)

Dadson et al.  
Nature 2003

Do steady state landscapes exist on Earth?



Erosion rate from sediment traps



Exhumation (“unroofing”) rate from apatite fission-track data

Fluvial sediment transport: what controls the shape of rivers?

BASIC EQUATIONS GOVERNING BED ELEVATION

**WHAT SETS RIVER PLANFORM?**

WHAT SETS RIVER LONG PROFILE?

BEDROCK RIVER EROSION

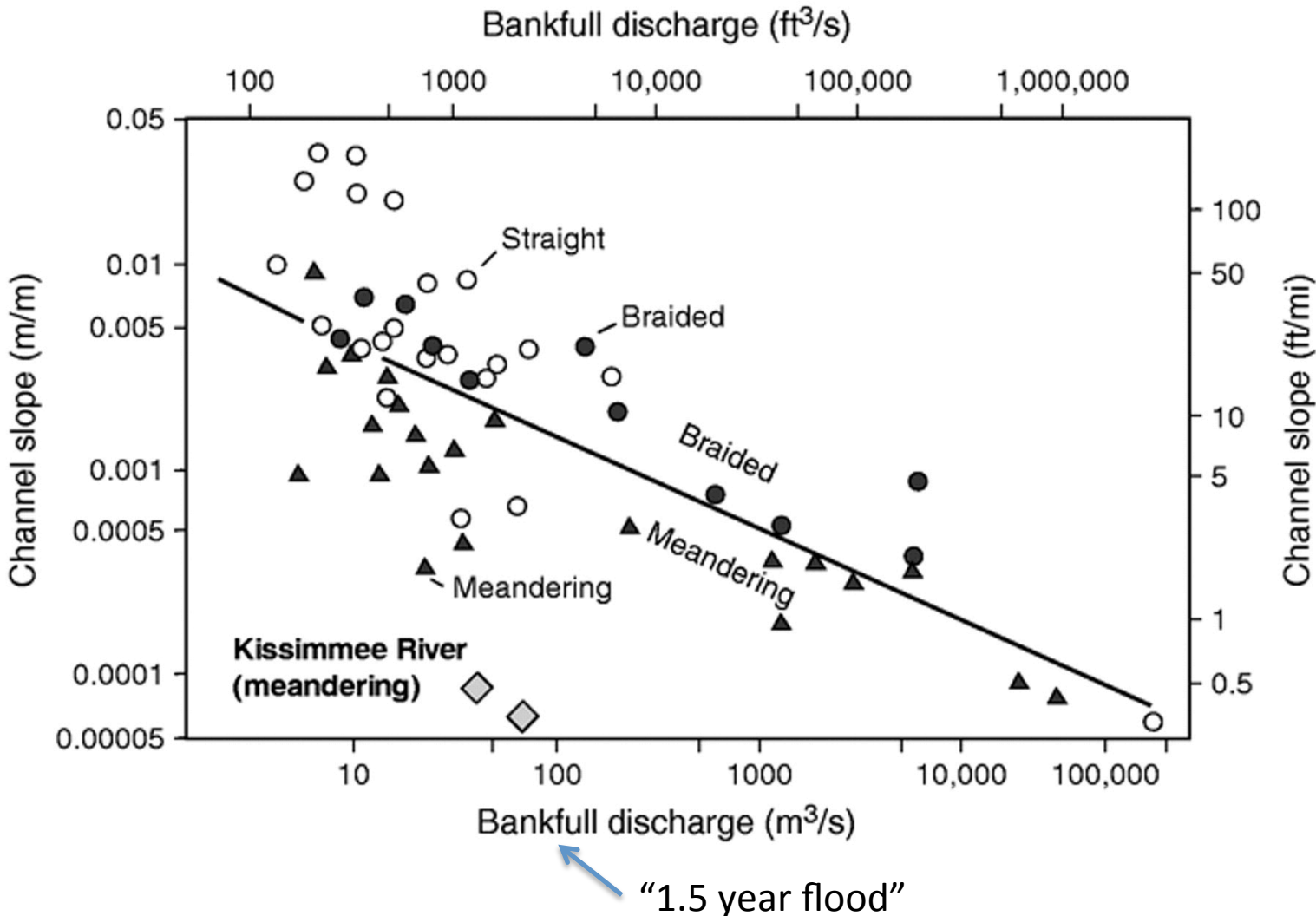
# Meandering outcompetes braiding when banks are strong

$$S_{\text{crit}} = 0.012 Q^{-0.44}$$

Possible causes:

A grainsize effect?

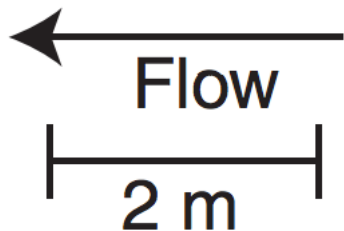
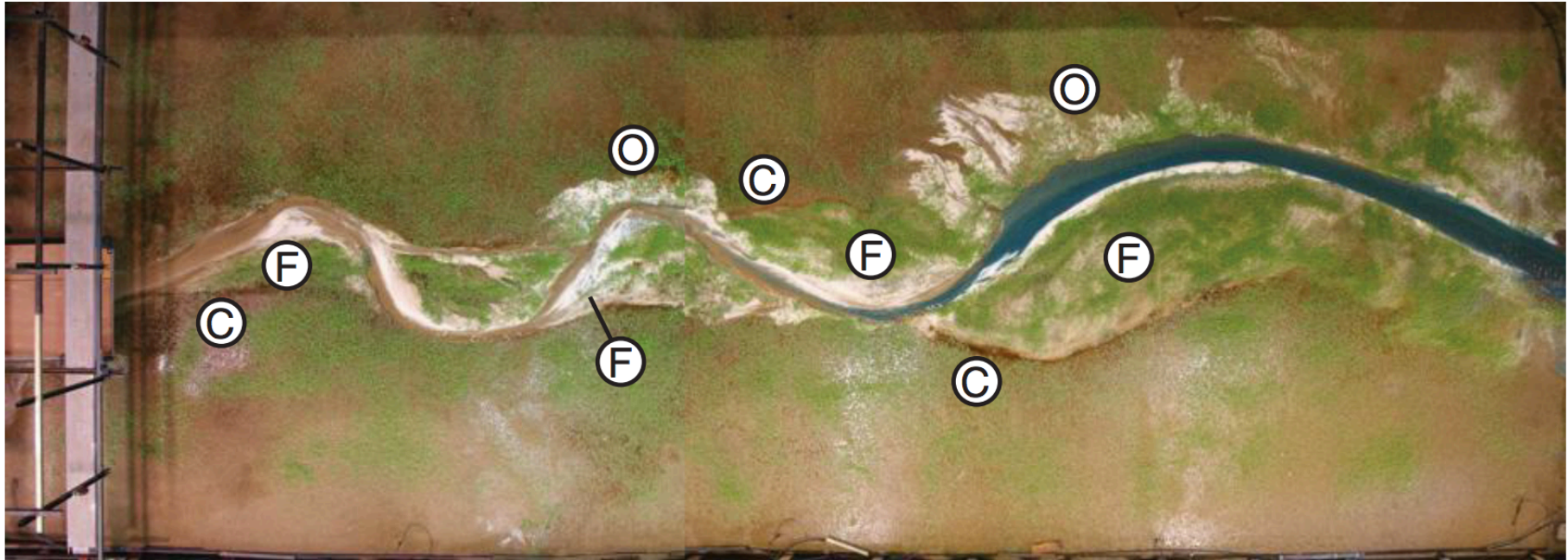
Due to weak channel banks?



Warne et al.  
GSA Bulletin 1999

# Laboratory replication of meandering is hard: vegetation matters

On Earth, most meandering-river deposits postdate the Devonian (evolution of roots)



- © Chute channel
- Ⓕ Fine sediment bar deposits
- Ⓞ Overbank deposits of fines



# Fluvial bedforms constrain paleo-depth of flow

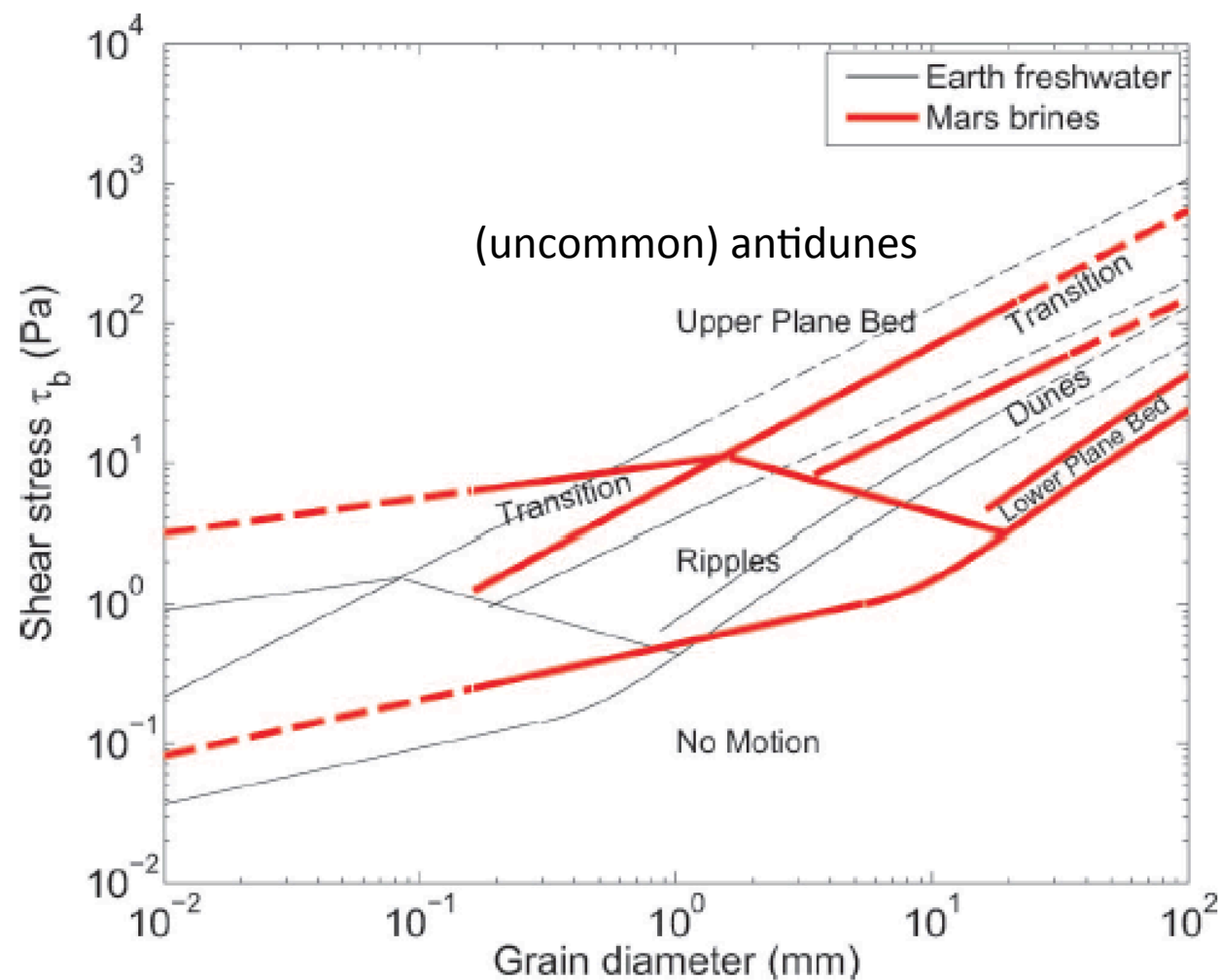


FIG. 7.—Translation of Figure 6 into dimensional space for the case of freshwater flows on Earth and a dense, viscous brine on Mars. See text for specific fluid properties. Note the dramatic shift in bed form space to coarser sediment for the brine case. Solid lines represent boundaries that have been explored experimentally following Figures 5 and 6, and dashed lines are extrapolations. Labels correspond to the Martian case.

Lamb et al. 2012, in  
“Sedimentary Geology of Mars,”  
Grotzinger & Milliken, eds.



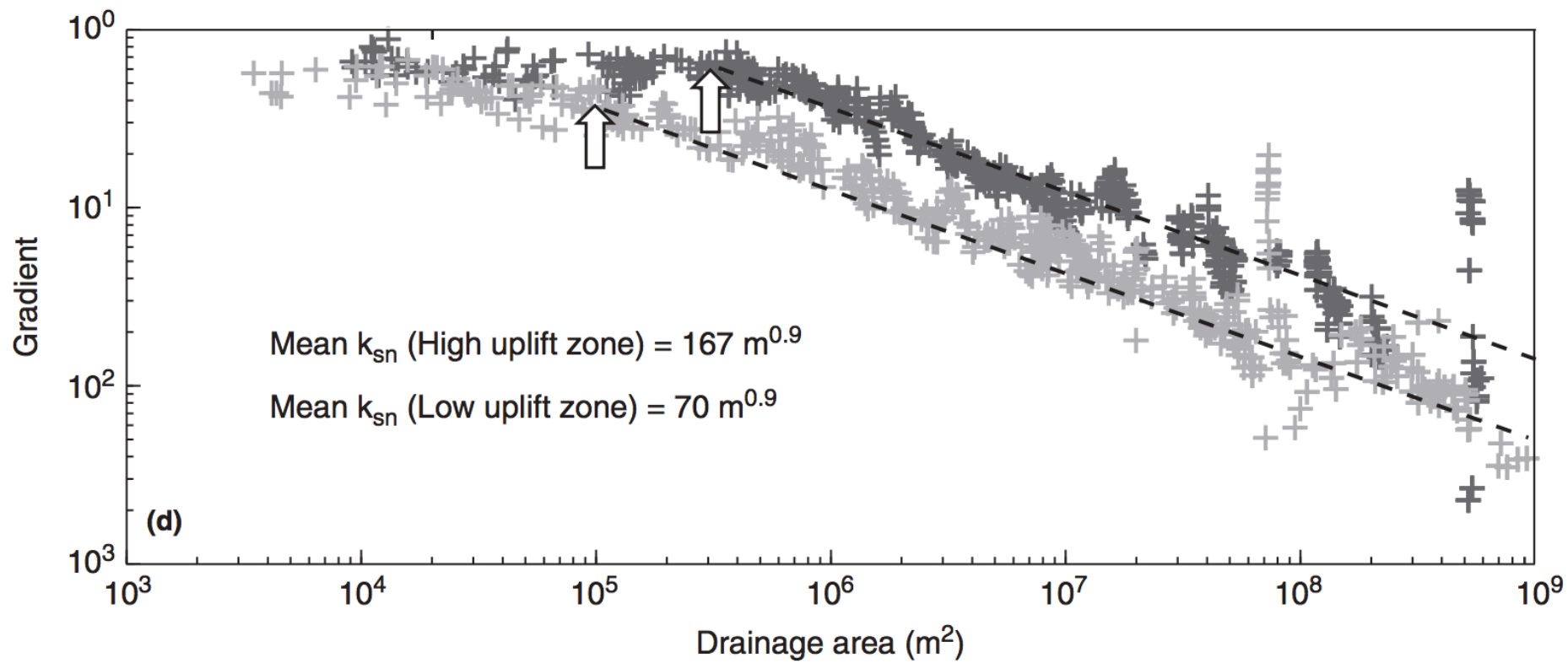
Fluvial sediment transport: what controls the shape of rivers?

BASIC EQUATIONS GOVERNING BED ELEVATION

WHAT SETS RIVER PLANFORM?

WHAT SETS RIVER LONG PROFILE?

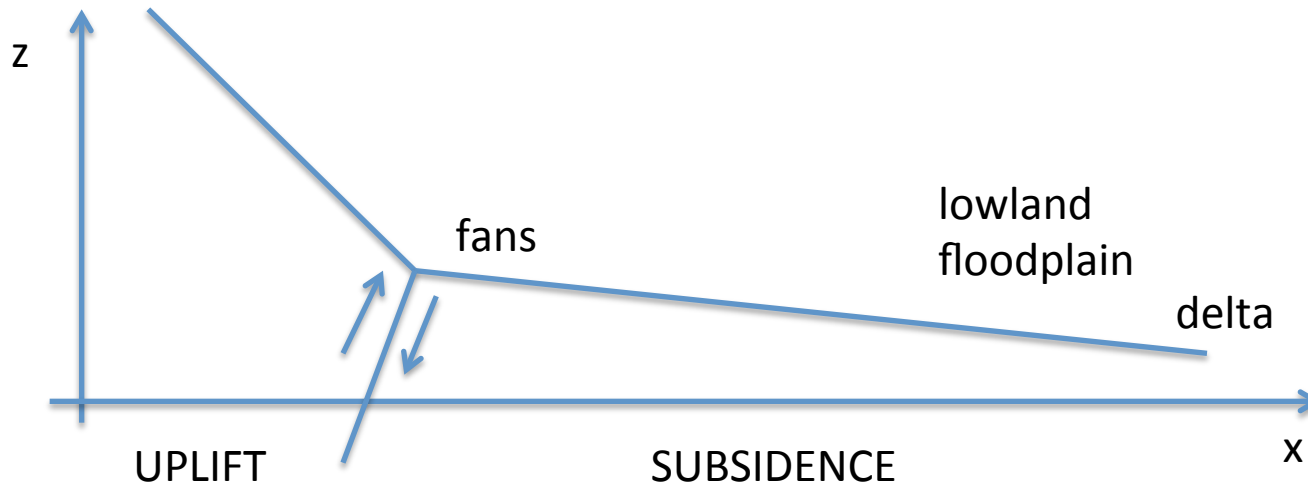
BEDROCK RIVER EROSION



Debris flows and hillslopes

Fluvial processes

The broadest possible view (**hypothesis 1**):



However, we find that concavity is found within the bedrock-cutting and sediment-transporting parts of the system. So what causes this concavity – if not the grand-scale tectonics?

First, assume  $\tau_b = \tau_c$  everywhere on the profile (just moving “indestructible balls”).

Then,  $\rho g h S = \tau_c$ . And  $\tau_c / (\rho_s - \rho) g D_{50} = \tau_{*c} = “k” \sim \text{constant}$  ( **$\sim 0.045$  for gravel, 0.03 for sand**).

$$\rightarrow S = k(\rho_s - \rho) g D_{50} / \rho g h$$

$$\rightarrow S = k_1 h^{-1} \quad (\text{hypothesis 2 ...})$$

From hydraulic geometry,  $h = cQ^f$ ,  $f \sim 0.4$ . (Definition:  $Q = Q_{bf}$  = bankfull discharge)  
*last lecture ...*  $Q$  is proportional to  $A$  (for small catchments)

$$S = k_2 A^{-0.4} D_{50} \quad (\text{hypothesis 2})$$

Prediction: A river's slope is to some extent controlled by the grainsize it receives.

**What if  $D_{50}$  changes (change in hillslope or tributary input, clast abrasion/dissolution)?**  
**Downstream fining - e.g. , breakdown of sediment along the stream path?**

Experimentally,  $D(x) = D_0 e^{-\beta x}$   $x$  is distance along flowpath  
"Sternberg's law"

with  $\beta = (10^{-5} - 10^{-1}) \text{ km}^{-1}$  (function of lithology)

Set  $L_{1/2}$  = travel distance at which grainsize is reduced by  $\frac{1}{2}$   
 $L_{1/2} = \ln(2)/\beta = (7 - 70,000) \text{ km}$

Set  $L_D^*$  = travel distance at which grain goes into suspension.  
Then  $L_D^* = (1/\beta) \ln (D_0/D_{\min})$ , where  $D_{\min}$  = size at suspension.

Amplitude of concavity depends on sediment supply  
(and sediment abrasion)

Transported sediment:

(hypothesis 3)

$q_s = k\tau_b^n$  per unit width (neglect  $\tau_c$ )

$Q_s = q_s w = k\tau_b^n w$ ;  $\tau_b = \rho ghS$ , where  $h = cA^\gamma$ ;  $w = dA^b$

Input sediment:  $Q_s = aA^\alpha$

$= k_1(\rho gcA^\gamma S)^n dA^b$

Conservation of mass:

$aA^\alpha = k_1(\rho gc)^n A^{\gamma n} S^n dA^b$

$S = k_2 A^{(\alpha - \gamma n - b)/n}$

**Examples:**

$\alpha = 1.0$ ,  $\gamma = 0.4$ ,  $n = 1.5$ ,  $b = 0.5$ :

$S = k_2 A^{-0.07} \rightarrow$  **straight profile.**

But, if  $\alpha = 0$ ,

$S = k_2 A^{-0.73} \rightarrow$  **large concavity.**

(downstream fining control = hypothesis 4)

For rivers that cut through bedrock, how does this erosion affect the long profile?

$dz/dt = U - E$ . Hypothesis:  $E = k(\text{streampower})^n$

**“Streampower”**: power = force x distance / time

$$\text{power}/(\text{bed area}) = \frac{F \times d}{A \times t}$$

$\tau_b$

river speed

**hypothesis:**

widely used &  
physically reasonable,  
but not tested enough  
to be trusted

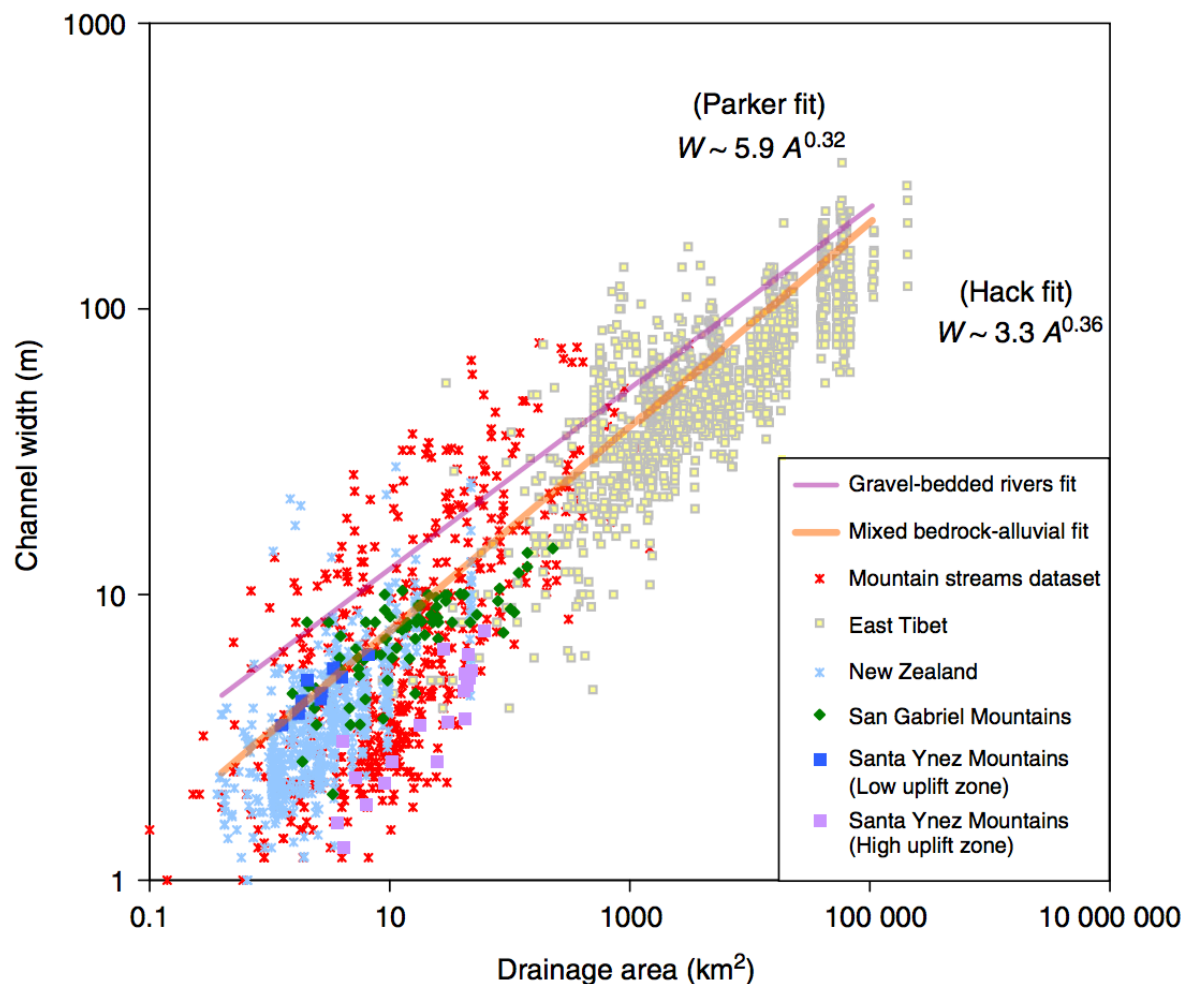
$$\omega = \tau_b V$$

$$Q_w = h v w \quad (Q_w = \text{water flux})$$

$$\omega = \rho g h V S = \rho g Q_w S / w$$

$$E = k (\rho g Q_w S / w)^n$$

Streampower erosion law



**Figure 2** Bedrock channel width as a function of upstream drainage area in graded bedrock rivers. Power-law scaling relations for alluvial gravel-bedded rivers (e.g., [Parker et al., 2007](#)) and mixed bedrock–alluvial rivers ([Hack, 1957](#)) are shown for comparison. Data includes rivers undergoing a wide range of uplift (and incision) rates from [Wohl and Merritts \(2005\)](#) global compilation, the eastern margin of the Tibetan Plateau ([Kirby and Ouimet, 2011](#)), New Zealand ([Crosby, 2006](#)), the San Gabriel Mountains ([DiBiase et al., 2009](#)), and the Santa Ynez mountains ([Duvall et al., 2004](#)). Only the Santa Ynez data show a narrowing of channels in zones of higher rock uplift in graded bedrock rivers. Albeit with considerable scatter, bedrock rivers show the same scaling with drainage area as gravel-bed alluvial rivers (which also show much scatter), with mean channel width slightly narrower in bedrock channels (less than a factor of 2) for the same drainage area. Some of the observed scatter is likely attributable to differences in runoff and flood variability among sites, differences in substrate properties, sediment load, and differences in bed morphology. Reproduced from Wohl, E., Merritt, D.M., 2008. Reach-scale channel geometry of mountain streams. *Geomorphology* 93(3–4), 168–185.

## Streampower - topographic steady state:

$$Q = k_q A^c \quad w = k_w Q^b \quad Q = v h W \quad \tau_b = \rho g h S \quad \omega = \tau_b v$$

$$E = (\rho g)^n (k_3^{(1-b)} k_4^{-1})^n A^{en(1-b)} S^n$$

Following Whipple and Tucker  
JGR 1999

define this as “m”

$$\rightarrow E = k_5 A^m S^n$$

→ The streampower hypothesis leads to  $E = k A^m S^n$

→ “m” contains channel geometry information.

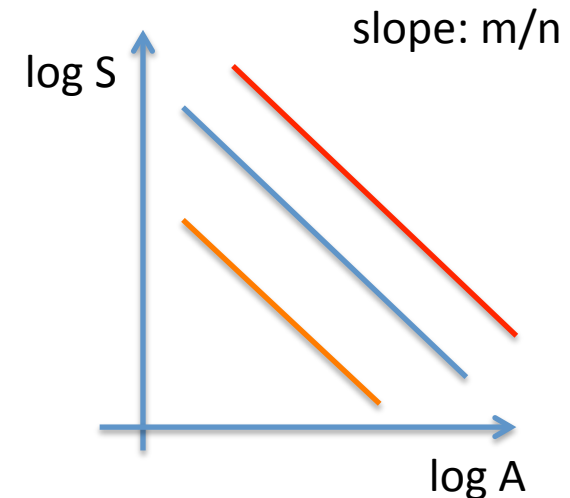
### Steady state:

$$dz/dt = U - E = 0$$

$$S = (U/k_5)^{1/n} A^{-m/n}$$

Concavity

(“Whipple’s”) steepness  
index




Increasing uplift or harder rock  
Decreasing uplift or softer rock



# Summing up controls on longitudinal profiles: hypotheses

- (1) Gross topography (set by tectonics)
- (2) Threshold rivers (intent: gravel-bed rivers)
- (3) Power-law relation between sediment transport flux and basal shear stress (intent: sand-bed rivers)
- (4) Selective transport
- (5) Bedrock incision in canyons:
  - (5a) Streampower
  - (5b) Tools-and-cover (Leonard Sklar)



Large boulders: effectively rough pieces of bedrock (abraded in place)  
*Sourcing* may change from coarse to fine downstream.

Fluvial sediment transport: what controls the shape of rivers?

BASIC EQUATIONS GOVERNING BED ELEVATION

WHAT SETS RIVER PLANFORM?

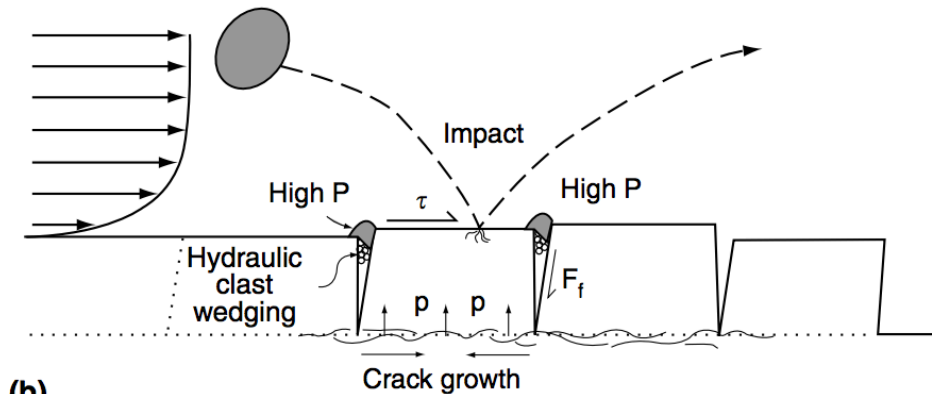
WHAT SETS RIVER LONG PROFILE?

**BEDROCK RIVER EROSION**

# Mechanisms of bedrock erosion: plucking



(a)

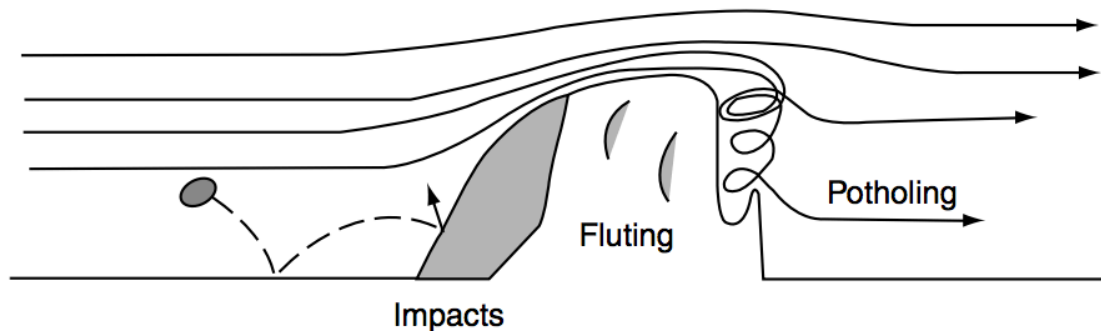


(b)

# Mechanisms of bedrock erosion: abrasion



(c)





# Mechanisms of bedrock erosion: corrosion (= weathering + dissolution)

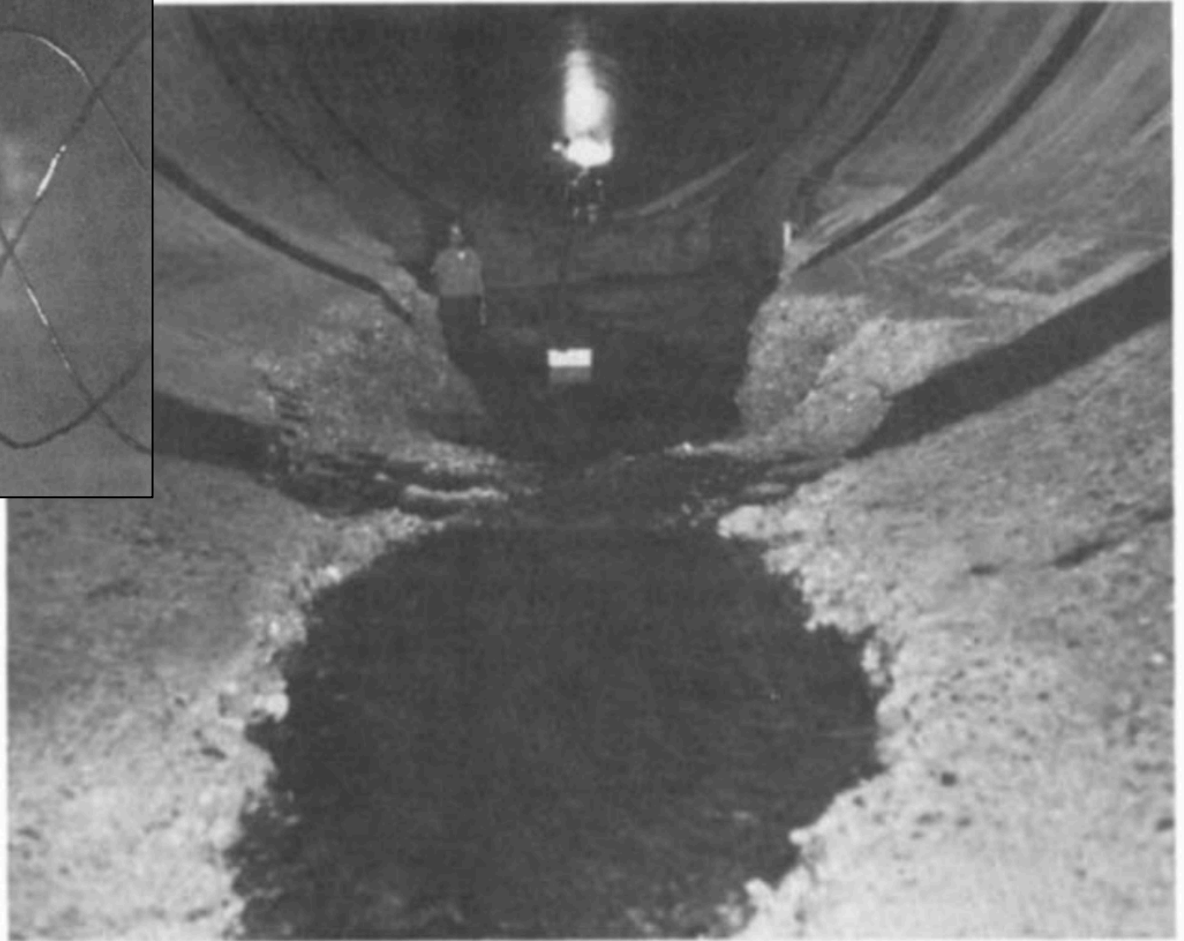
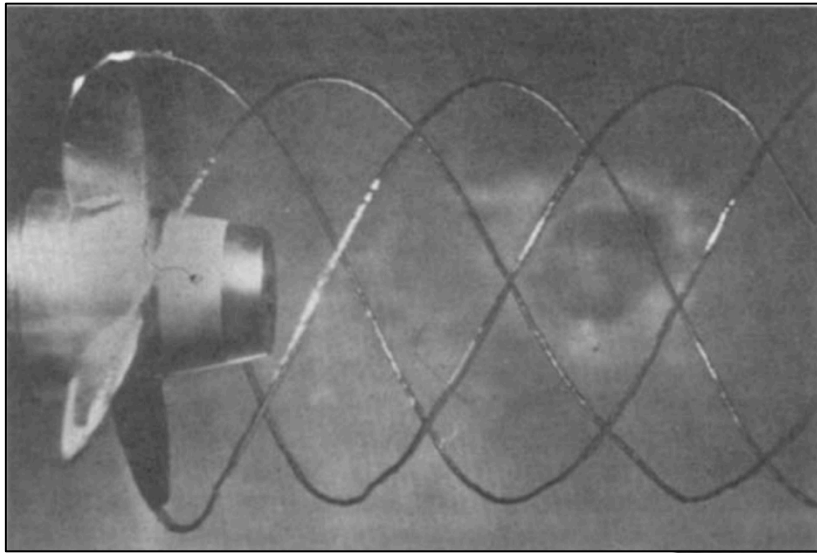


**Tsingy de Bamaraha, Madagascar  
(UNESCO World Heritage site)**



**Lighthouse Reef Atoll Blue Hole, Belize.**

# Mechanisms of bedrock erosion (?): cavitation?



**Figure 1** Cavitation damage in the spillway tunnel of the Yellowtail Dam in Montana. Courtesy of US Bureau of Reclamation.

Points to remember for lab:

# Ten Commandments of Landscape Evolution Modeling

1. Thou shalt not use a model without understanding the ingredients therein.
2. Be thou ever mindful of uncertainty.
3. Thou shalt use thy model to develop insight.
4. Thou shalt take delight when thy model surprises thee.
5. Thou shalt kick thy model hard, that it may notice thee (an injunction borrowed gratefully from the 10 Climate Modeling Commandments).
6. Thou shalt diagnose the reasons for thy model's behavior.
7. Thou shalt conduct sensitivity experiments and “play around.”
8. Thou shalt use thy model to discover the necessary and sufficient conditions needed to explain thy target problem.
9. If thou darest use a model to calculate what happened in your field area in the past, thou shalt find a way to test and calibrate it first.
10. If thou darest to predict future erosion, thou shalt heed the previous commandment ten times over (but thou mightest point out to skeptics that a process-based prediction is usually better than one based on pure guesswork, provided that commandment #2 is obeyed).

(Greg Tucker)



# Key points from today's lecture

- Advective vs. diffusive channel profile evolution
- Hypotheses for controls on concavity
- Know the streampower equation
- Know bedrock erosion processes

*In the required reading, don't omit the section on transient evolution (bridge to next lecture)*

*Next lecture: Landscape-scale responses to forcing*