GEOS 28600

The science of landscapes:
Earth & Planetary Surface Processes

http://geosci.uchicago.edu/~kite/geos28600_2019/

Lecture 8

Monday 11 Feb 2019

Fluvial sediment transport, continued

Logistics

- Homework 3 is due now
- Lab tomorrow 11a-noon, Hinds 440
- Homework 4 will be issued tonight

Fluvial sediment transport: introduction

TURBULENT VELOCITY PROFILES, INITIATION OF MOTION

BEDLOAD, RIVER GEOMETRY

Key points from “Introduction to fluvial sediment transport”:

- “Law of the wall” – how to calculate river discharge from elementary measurements (bed grain size and river depth).

- Critical Shields stress
- Differences between gravel-bed vs. sand-bed rivers
- Discharge-width scaling

Fluvial sediment transport: introduction

TURBULENT VELOCITY PROFILES, INITIATION OF MOTION

BEDLOAD, RIVER GEOMETRY

Hydraulics and sediment transport in rivers:

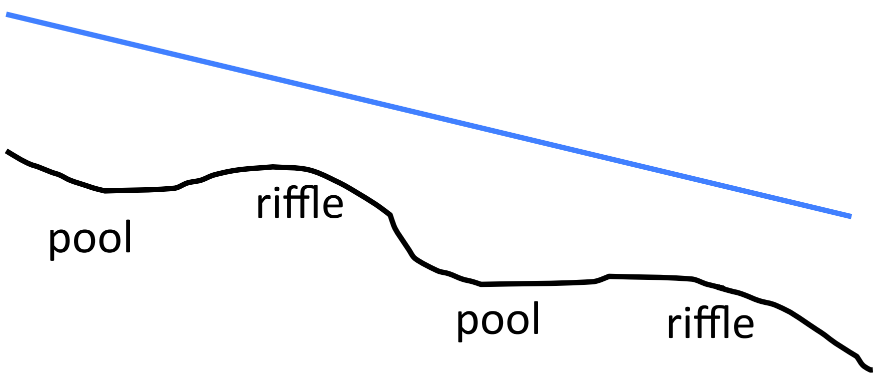
1) Relate flow to frictional resistance so can relate discharge to hydraulic geometry.

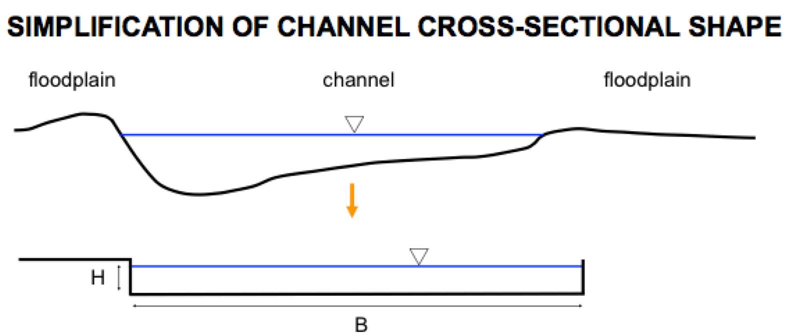
2) Calculate the boundary shear stress.

Simplified geometry: average over a reach (12-15 channel widths).

→ we can assume accelerations are zero.

→ this assumption is better for flood flow (when most of the erosion occurs)





Parker Morphodynamics e-book

The assumption of no acceleration requires that gravity (resolved downslope) balances bed friction.

$\tau_{zx} = \rho g h \sin\theta$

averaging over 15-20 channel widths forces the water slope to ~ parallel the basal slope

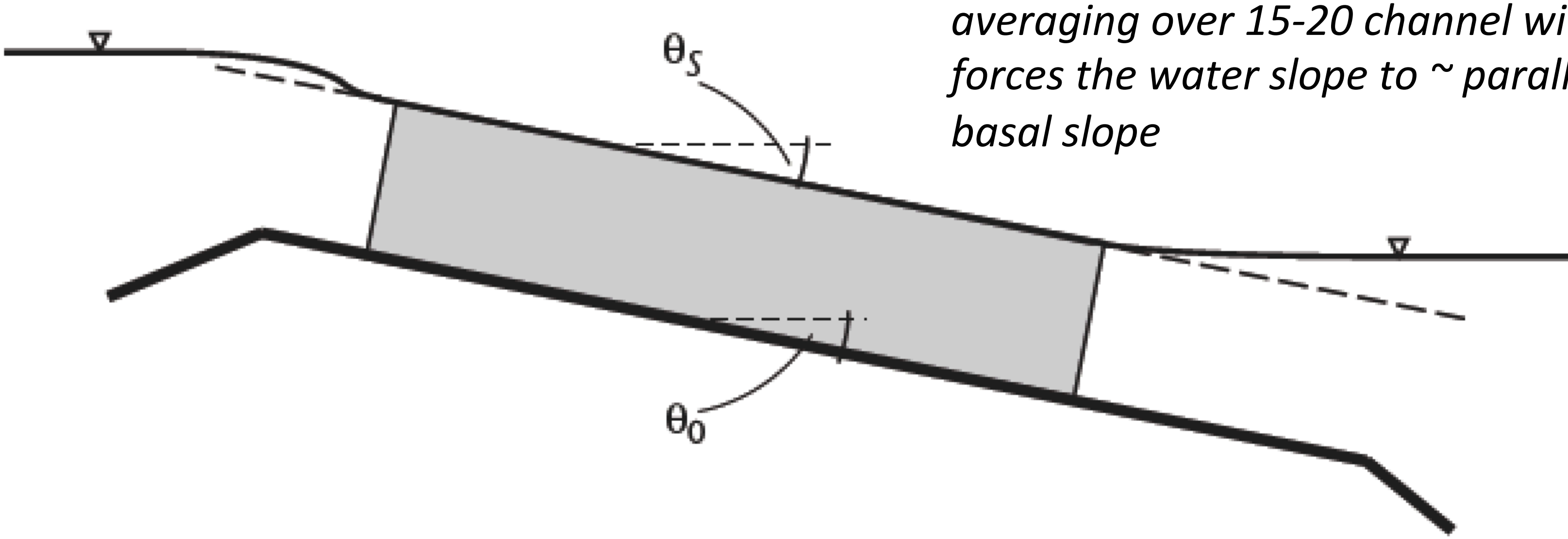
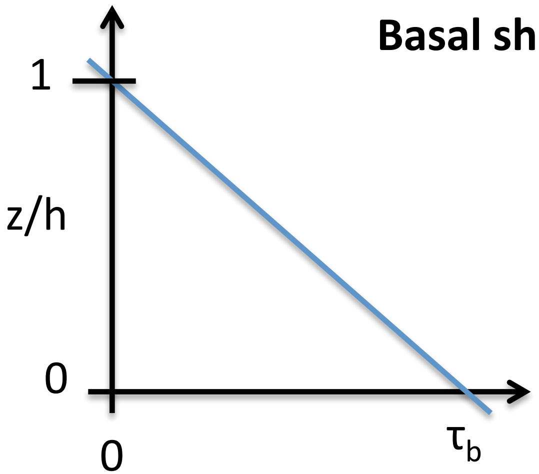


Figure 6.2 Idealized development of uniform flow in a channel of constant slope, θ_0, geometry, and bed material connecting two reservoirs. The shaded area is the region of uniform flow, where the downstream component of gravity is balanced by frictional resistance and the water-surface slope θ_S equals θ_0.

Dingman, chapter 6

Basal shear stress, frictional resistance, and hydraulic radius

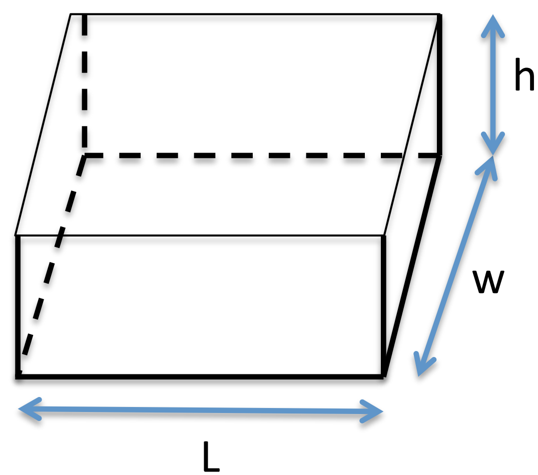


$\tau_{zx} = \rho gh \sin\theta$

At low slope (S, water surface rise/run), $\theta \sim \tan\theta \sim \sin\theta$

$\tau_b = \rho gh\, S$

<u>Frictional resistance:</u>



Boundary stress = $\rho gh \sin\theta\, L\, w$

Frictional resistance = $\tau_b\, L\, (w + 2\, h)$

$\rho gh \sin\theta\, L\, w = \tau_b\, L\, (w + 2\, h)$

→ $\tau_b = \rho gh\, (\, w / (w + 2\, h\,)\,) \sin\theta$

Define hydraulic radius, $R = hw / (w + 2\, h)$

→ $\tau_b = \rho gR \sin\theta$

In very wide channels, R → h (w >> h)

Law of the wall:

Rivers (Re >> 10, and fully turbulent):

$\tau_{zx} = \rho K_T (du/dz)$

eddy viscosity, "diffuses" velocity

Glaciers (Re << 1):

$\tau_{zx} = \mu(T,\sigma) (du/dz)$

Properties of turbulence:
Irregularity
Diffusivity
Vorticity
Dissipation

From empirical & theoretical studies:

$K_T = (k z)^2 (du/dz)$ *(where k = 0.39-0.4 = von Karman's constant)*

→ $\tau_B = \rho(k z)^2 (du/dz)^2$

→ $(\tau_B / \rho)^{1/2} = k z (du / dz) = u^*$ = "shear velocity"

Memorize this.

→ $(\rho g h S / \rho)^{1/2} = u^* = (g h S)^{1/2}$

Now $u^* = k z (du / dz)$

Separate variables: $du = (u^* / k z) dz$

Integrate: $u = (u^*/k) (\ln z + c)$. For convenience, set $c = -\ln(z_0)$

(explained on next slide)

"law of the wall"

Then, $u = (u^*/k) \ln (z/z_0)$

when $z = z_0$, $u = 0$ m/s.

Calculating river discharge, Q (m^3s^{-1}), from elementary observations (bed grain size and river depth).

$$u = (u^*/k) \ln (z/z_0)$$

"law of the wall"

z_0 is a length scale for grain roughness varies with the size of the bedload. In this class, use $z_0 = 0.12\ D_{84}$, where D_{84} is the 84th percentile size in a pebble-count (100th percentile is the biggest).

$$Q = \langle u \rangle\, w\, h$$

brackets denote vertical average

$$\langle u \rangle = \int_{z_0}^{h} u(z)\, dz \quad (1/(h-z_0))$$

$$\langle u \rangle = (u^*/k)\,(z0 + h\,(\ln(h/z_0) - 1))\,(1/(h - z_0))$$

$h >> z_0$:

$$\langle u \rangle = (u^*/k)\quad(\ln(h/z_0) - 1)$$

$$\langle u \rangle = (u^*/k) \ln (h / e\, z_0)$$

$$\langle u \rangle = (u^*/k) \ln (0.368\, h / z_0)$$

typically rounded to 0.4

Extending the law of the wall throughout the entire depth of the flow is a rough approximation – do not use this for civil-engineering applications. This approach does not work at all when depth → clast grainsize.

Drag coefficient for bed particles:

3 alternative methods

→ $\tau_B = \rho g R S = C_D \rho \langle u \rangle^2 / 2$

$\langle u \rangle = (2g R S / C_D)^{1/2}$ $(2g / C_D)^{1/2} = C$ = Chezy coefficient

$\langle u \rangle = C (R S)^{1/2}$ Chezy equation (1769)

$\langle u \rangle = (8 g / f)^{1/2} (R S)^{1/2}$ f = Darcy-Weisbach friction factor

---

$\langle u \rangle = R^{2/3} S^{1/2} n^{-1}$ n = Manning roughness coefficient

Most used, because lots of investment in measuring *n* for different objects

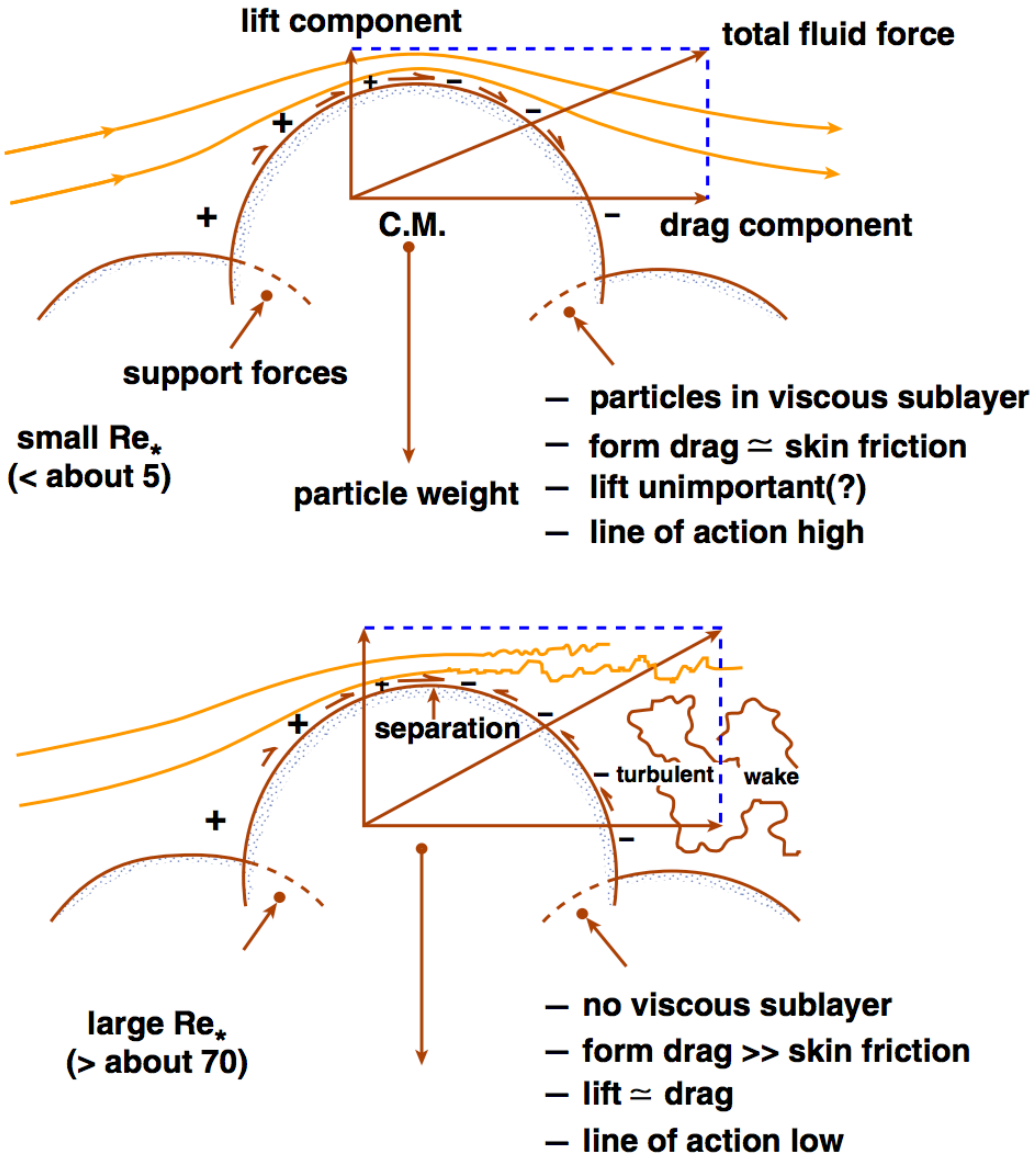
$0.025 < n < 0.03$ ----- Clean, straight rivers (no debris or wood in channel)
$0.033 < n < 0.03$ ----- Winding rivers with pools and riffles
$0.075 < n < 0.15$ ----- Weedy, winding and overgrown rivers
$n = 0.031(D_{84})^{1/6}$ ---- Straight, gravelled rivers

In sand-bedded rivers (e.g. Mississippi), form drag due to sand dunes is important.

In very steep streams, supercritical flow may occur: *supercritical flow*

Froude number $Fr \# = \langle u \rangle/(gh)^{1/2} > 1$

Getting from water flow to sediment flux



John Southard

Sediment transport in rivers: (Shields number)

At the initiation of grain motion,

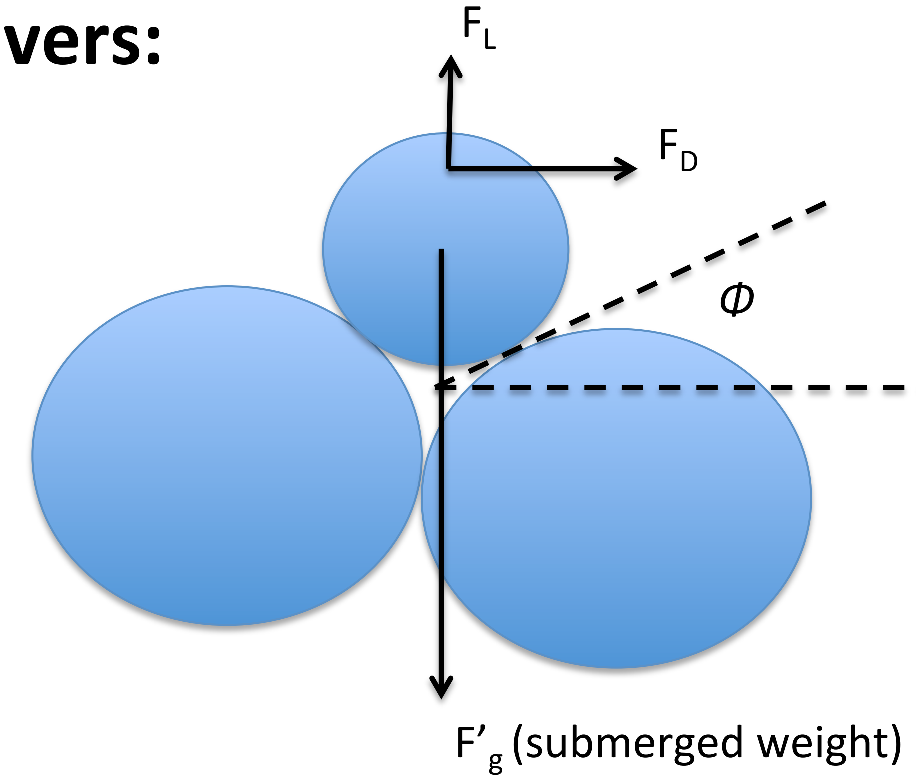
$$F_D = (F'_g - F_L) \tan \Phi$$

$$\rightarrow F_D/F'_g = \frac{\tan \Phi}{1 + (F_L/F_D) \tan \Phi}$$

$$\approx \frac{\tau_c D^2}{(\rho_s - \rho)gD^3}$$

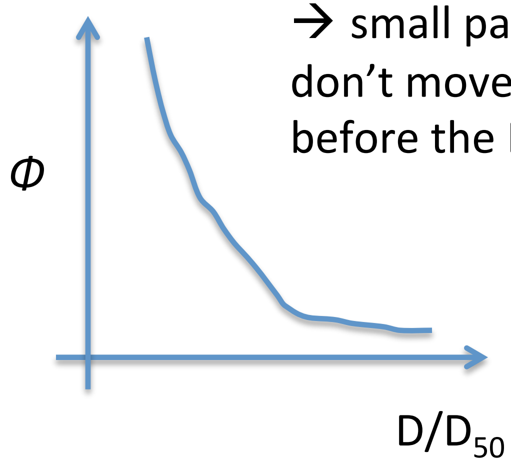
$$= \frac{\tau_c}{(\rho_s - \rho)gD} = \tau_*$$

Shields number ("drag/weight ratio")



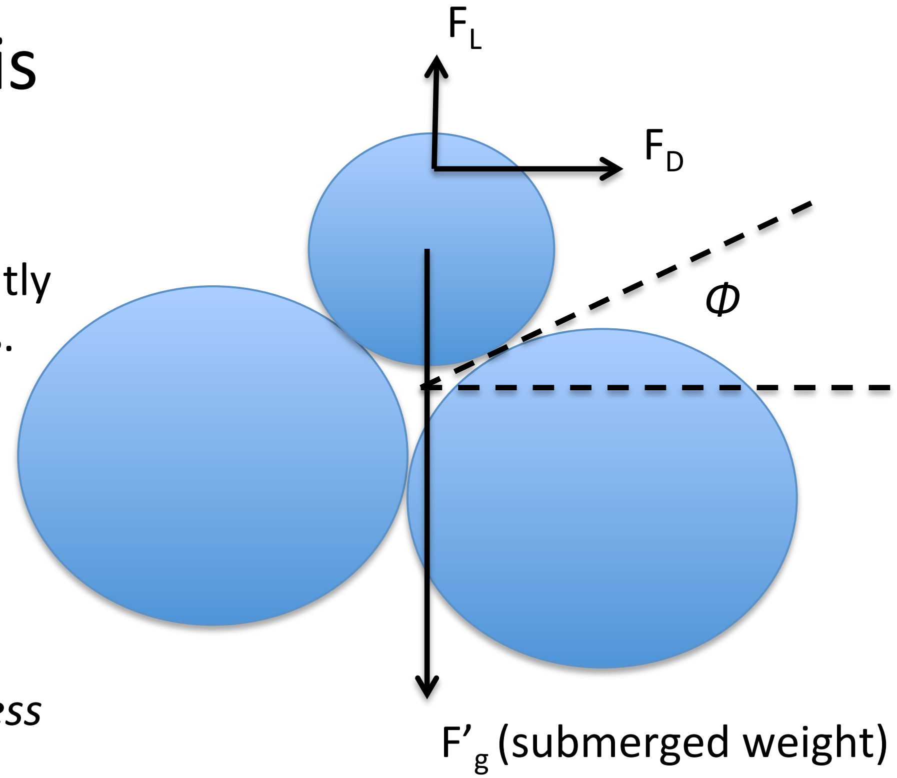
Is there a representative particle size for the bedload as a whole?
Yes: it's D_{50}.

Equal mobility hypothesis



"Hiding" effect
→ small particles
don't move significantly
before the D_{50} moves.

Trade-off between size and embeddedness



Significant controversy over validity of equal mobility hypothesis in the late '80s – early '90s.
Parameterise using

$$\tau_* = B(D/D_{50})^{\alpha}$$

α = -1 would indicate perfect equal mobility (**no** sorting by grain size with downstream distance)
α = -0.9 found from flume experiments (permitting long-distance sorting by grain size).

τ_{*c50} ~ 0.04, from experiments (0.045-0.047 for gravel, 0.03 for sand)

1936:

Hydraulically rough: viscous sublayer is a thin skin around the particles.

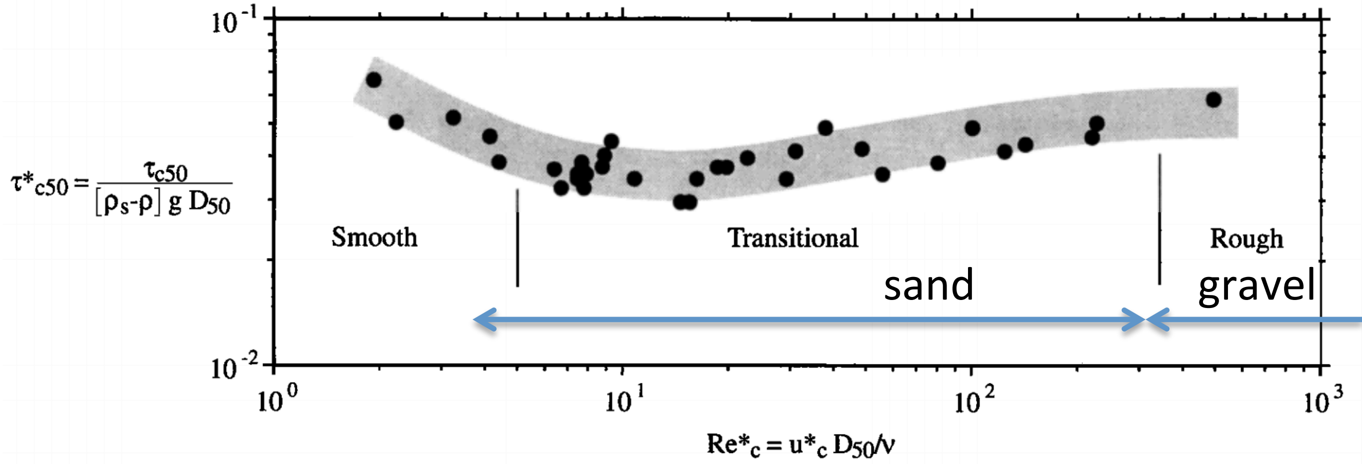
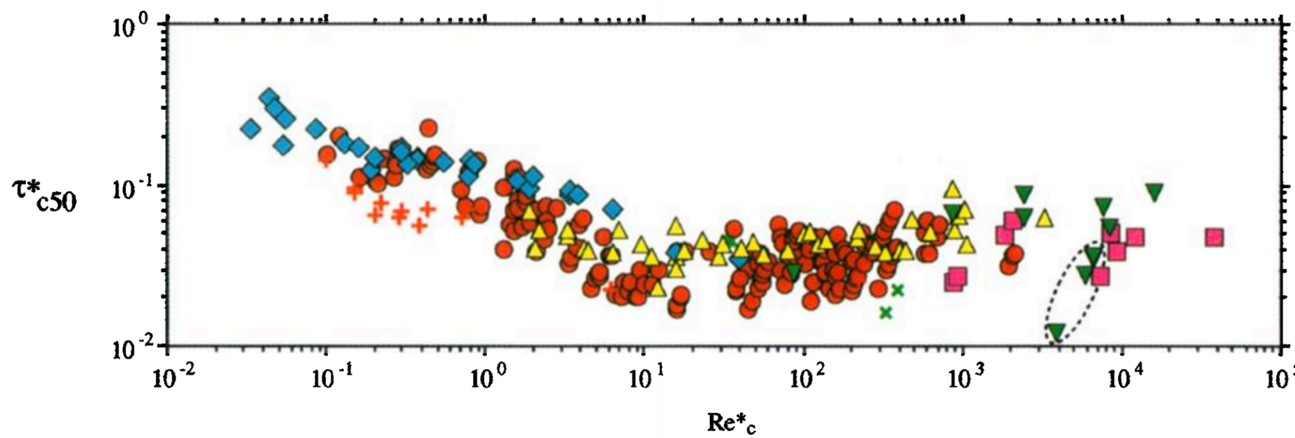


Figure 1. *Shields'* [1936] curve redrafted from *Rouse* [1939].

Re* = "Reynolds roughness number"

1999:

Theory has approximately reproduced some parts of this curve.



Causes of scatter:
(1) differing definitions of initiation of motion (most important).
(2) slope-dependence? (Lamb et al. JGR 2008)

Buffington & Montgomery, Water Resources Research, 1999

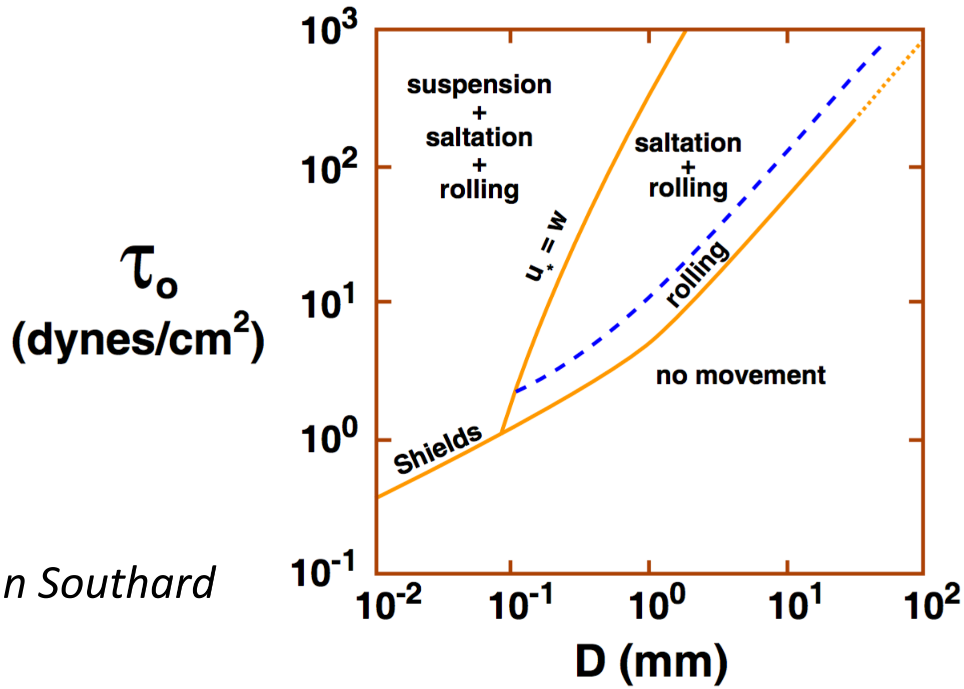
Fluvial sediment transport: introduction

REVIEW OF REQUIRED READING (SCHOOF & HEWITT 2013)

TURBULENT VELOCITY PROFILES, INITIATION OF MOTION

BEDLOAD, RIVER GEOMETRY

Consequences of increasing shear stress: gravel-bed vs. sand-bed rivers



John Southard

Suspension: characteristic velocity for turbulent fluctuations (u*) exceeds settling velocity (ratio is ~Rouse number).

Typical transport distance 100m/yr in gravel-bedded bedload
Sand: km/day

(Experimentally, u* is approximately equal to rms fluctuations in vertical turbulent velocity)

Empirically, rivers are either gravel-bedded or sand-bedded (little in between)
The cause is unsettled: e.g. Jerolmack & Brzinski Geology 2010 vs. Lamb & Venditti GRL 2016

Bedload transport

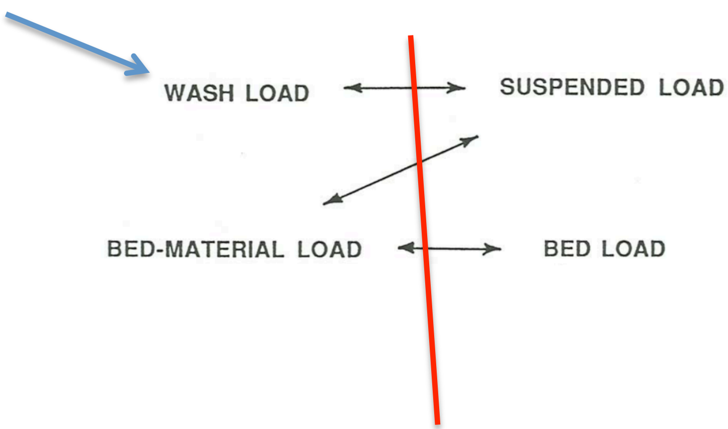
(Most common:)

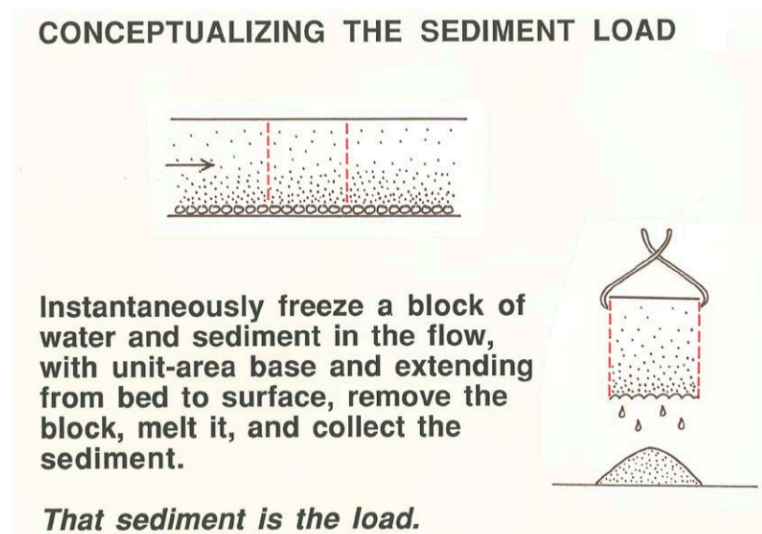
Meyer-Peter Muller

$$q_{bl} = k_b(\tau_b - \tau_c)^{3/2}$$

Many alternatives, e.g.
- Yalin
- Einstein
- Discrete element modeling

there is no theory for washload:
it is entirely controlled by upstream supply



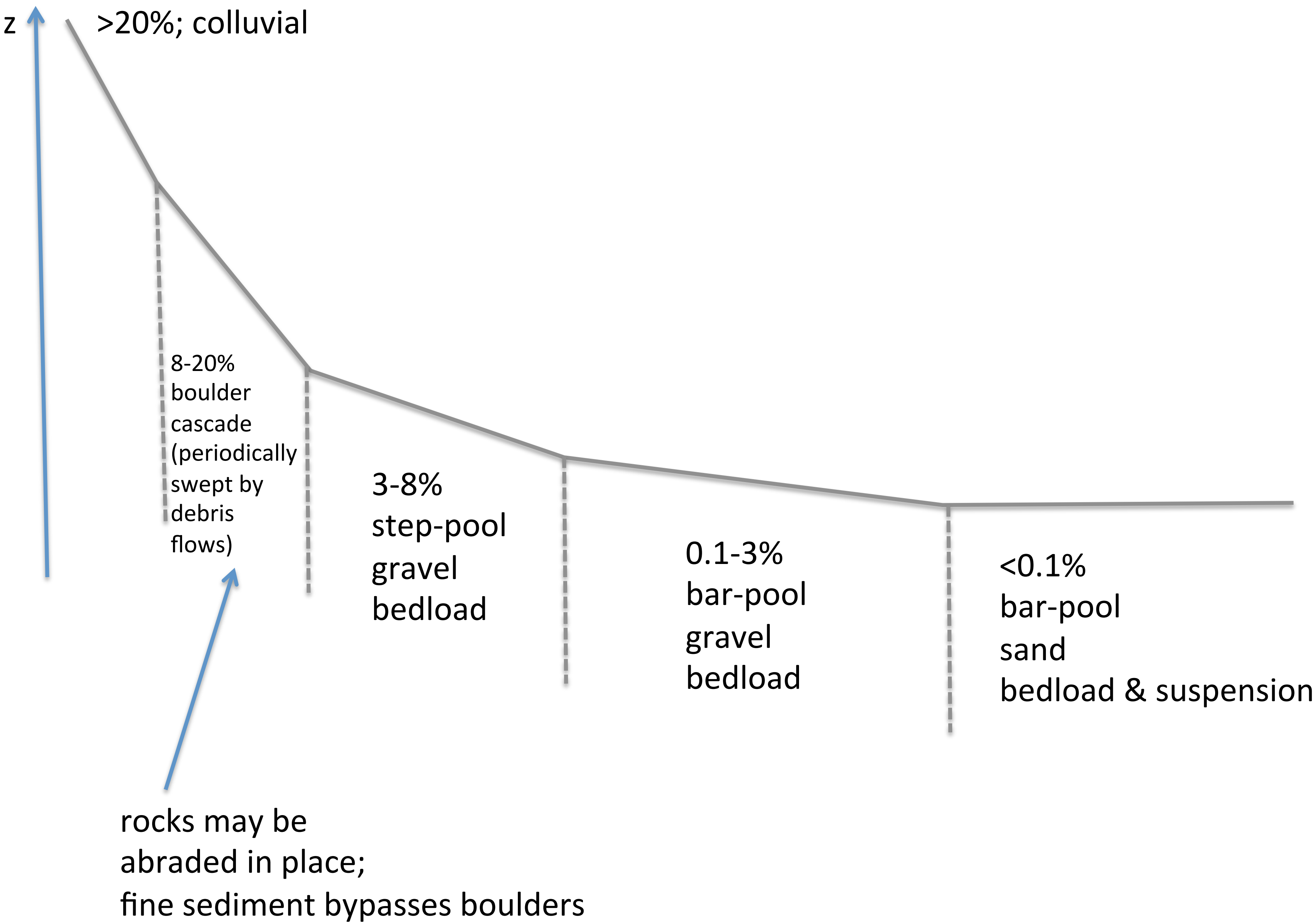


John Southard

River channel morphology and dynamics

- "Rivers are the authors of their own geometry" (L. Leopold)
  - *And of their own bed grain-size distribution.*
- Rivers have well-defined banks.
  - *Bankfull discharge 5-7 days per year; floodplains inundated every 1-2 years.*
  - *Regular geometry also applicable to canyon rivers.*
  - *Width scales as* $Q^{0.5}$
- River beds are (usually) not flat.
  - *Plane beds are uncommon. Bars and pools, spacing = 5.4x width.*
- Rivers meander.
  - *Wavelength ~ 11x channel width.*
- River profiles are concave-up.
  - *Grainsize also decreases downstream.*

Slope, grain size, and transport mechanism: strongly correlated



What sets width?

Q = wd<u>

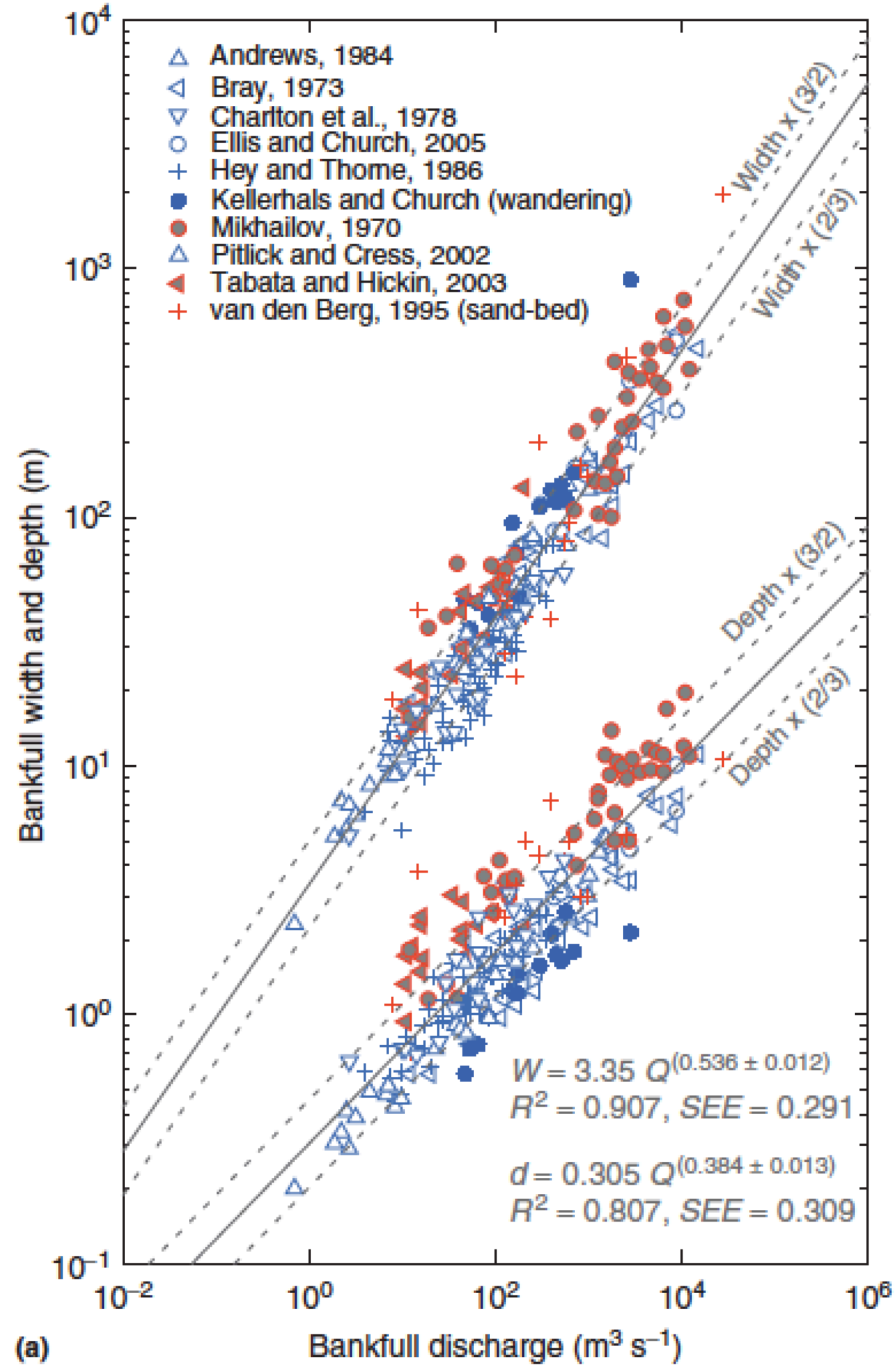
w = aQ^b

d = cQ^f

<u> = kQ^m

b+f+m = 1

Comparing different points downstream → b = 0.5, m = 0.1, f = 0.4



Eaton, Treatise on Geomorphology, 2013

What sets width? Three approaches to this unsolved question:

(1) Posit **empirical relationships between hydraulics, sediment supply, and form** (Parker et al. 2008 in suggested reading; Ikeda et al. 1988 Water Resources Research).

(2) **Extremal hypotheses**; posit an optimum channel, minimizing energy (Examples: minimum streampower per unit length; maximum friction; maximum sediment transport rate; minimum total streampower; minimize Froude number)

(3) What is the actual mechanism? What controls what sediment does, how high the bank is, & c.?

Key points from "Introduction to fluvial sediment transport"

- Law of the wall – how to calculate river discharge from elementary measurements (bed grain size and river depth).
- Critical Shields stress
- Differences between gravel-bed vs. sand-bed rivers
- Discharge-width scaling